Stability of Social Preferences and Learning in Public Goods Games

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Abstract
We examine the stability of social preferences in linear public goods games across different costs to cooperate and duration of interaction. Using laboratory experiments, we collect decisions from the same individual on conditional contributions in one-shot games and contributions in finitely (or indefinitely) repeated games at different relative costs of cooperation (i.e. the marginal per capita return (MPCR) to the public good). The generated data are used in a structural estimation of social preferences that accounts for decision error and cooperation costs. We find that, while the cost to cooperate changes behavior in predictable ways (e.g. when it is less costly to cooperate, individuals cooperate more), it also significantly changes the underlying distribution of estimated social preferences. That is, preferences are not stable as cooperation costs change. However, preferences are identical across one-shot and repeated games, and we do not find a significant effect of the type of repetition, finite or indefinite, on belief formation or social preferences. Contribution dynamics in the repeated game are best explained by a payoff-based reinforcement learning model, rather than a belief learning model with social preferences. Finally, using simulations, we show that for repeated games social preferences matter only insofar as they determine first round contributions and thereafter decisions are consistent with payoff-based reinforcement learning.

JEL Codes: H41, C92, D83, C63
Keywords: public goods games, social preferences, conditional cooperation, learning

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1 Introduction

The private provision of public goods and cooperation, more generally, may occur under a variety of institutional environments that differ in the costs to cooperate and duration of the interaction. Some environments may be more conducive than others to elicit and sustain cooperation. If it is costly to contribute to public goods, individuals may be less likely to do so, however, knowing that future interactions with others in a group are likely may induce higher contributions. Policies or mechanisms aimed at affecting cooperation levels may be desired but would face design challenges if both choices and preferences change across environments. Preference stability is an underlying assumption when designing incentives to promote cooperation, and changing preferences would present a moving target for design and could yield less effective, or ineffective, policies. Thus, it would be important to understand if and when cooperative preferences are stable, especially as the cost to cooperate changes and the time horizon of interactions differs.

The experimental evidence for stable cooperative preferences across varying costs and environments is mixed. In a one-shot environment, Volk, Thöni, and Ruigrok (2012) repeatedly elicited conditional contribution choices in a fixed subject pool over a period of five months for one cost to cooperate. They find stability of conditional preferences for cooperation at the aggregate level but a considerable amount of instability at the individual level. Cartwright and Lovett (2014) show evidence for aggregate level stability of conditional preferences as the cost of cooperation changes, and some studies find individuals become more selfish over time even when the opportunity to learn is absent (Brosig, Riechmann, & Weimann, 2007; Sass & Weimann, 2012). The results from Fischbacher and Gächter (2010) and Fischbacher, Gächter, and Quercia (2012) indicate that conditional contribution choices elicited from one-shot games do not explain a significant portion of the variance in behavior in repeated public good games. Finally, evidence from finitely and indefinitely repeated public good games shows no significant differences in contribution behavior (Tan & Wei, 2014; Lugovskyy et al., 2015). Identification of underlying preferences is complex, and stability in cooperative preferences is usually inferred from conditional contribution decisions. Decisions may change across environments, but it is unclear if variation in choices reflect unstable fundamental social preferences or not (Fehr & Schmidt, 1999; Bolton & Ockenfels, 2000; Charness & Rabin, 2002; Cox & Sadiraj, 2007; Cox, Friedman, & Sadiraj, 2008; Arifovic & Ledyard, 2012).

We examine the stability of social preferences across one-shot and repeated environments as cooperation costs change with data generated from controlled laboratory experiments. To help identify preferences based on observed choices, taking into account decision error, we use structural econometric analysis based on a social preference model that accommodates cost changes. We also examine learning and belief updating under finite and indefinite repetition. This approach allows us to address three questions. First, are underlying social preferences stable as the cost to cooperate changes? Second, keeping the costs to cooperate fixed, do underlying preferences change across one-shot and repeated interaction environments? Third, does the type of repetition (finite and indefinite) affect social preferences in a systematic manner?
The experiments are based on Fischbacher and Gächter (2010) and designed to observe behavior of the same individual in four linear public goods game settings. These are a one-shot conditional contribution game (labeled the P-task) and a repeated game with partners (labeled the R-task) at two different costs to cooperate (i.e. MPCR levels), one low and one high, that preserve the social dilemma. The one-shot game uses the strategy method to elicit an unconditional contribution and conditional contributions based on several possible average contributions of the other group members. Each one-shot game is then followed by a repeated game at the same MPCR level. In this game, along with deciding on a contribution in each round, we elicited beliefs about the average contribution of the other group members. The elicited beliefs allow for an examination of belief learning over rounds. The one-shot and repeated game sequence is then conducted again at a different MPCR level. In addition, across sessions but not within sessions, we varied whether the repeated games were finite or indefinite. Finally, the order of which MPCR is seen first is also reversed across sessions to control for experience effects.

Our design affords the opportunity to observe the decisions of the same individual at different costs to cooperate and in interactions of varying duration. The structural preference parameter estimates allow us to explore whether the distribution of preferences is the same as the environment changes from one-shot to repeated, as the costs to cooperate change, and as repetition changes from finite to indefinite. Since behavior under different costs and interaction duration should capture the same underlying preferences of the population, we can separate preferences from biases introduced by the cost and length of interaction.\footnote{The type of repetition (finite or indefinite) is randomly assigned to sessions, so the effect of the type of repetition on preferences is theoretically identified.} Using our approach, we can also identify which environments are more prone to generate noisy behavior.

One of the main innovations of our work is the structural estimation of altruism and one-sided fairness preferences separate from decision error. Much of the research on social preferences either assumes no error or does not explicitly model it at the individual level. If individuals do make errors, then not accounting for these in the estimations would produce biased results. The bias could be particularly pronounced in environments such as linear public goods games where both pure self-interested behavior and full cooperation would be underestimated because any deviation from these two extremes (zero and full contribution) would be interpreted as support against those behaviors. Evidence of this issue is already well known in the literature on linear public goods games (see Andreoni (1995); Houser and Kurzban (2002); Burton-Chellew and West (2013); Kümmerli, Burton-Chellew, Ross-Gillespie, and West (2010)). We incorporate heterogeneity in the tendency to make random optimization errors in our estimations, and this allows for cleaner estimates of social preferences. The random coefficients model we employ in our estimations also allows us to compute “posterior” estimates of the social preference parameters and decision error parameter by conditioning on individual level choices. We use these individual level estimates to evaluate the in-sample and out-of-sample predictive success of these parameters in explaining repeated game choices.\footnote{Our work can be viewed as building on that of Blanco, Engelmann, and Normann (2011).}
Using the estimates from the aggregate and individual-level data, we find mixed evidence for the stability of social preferences across the cost to cooperate and interaction duration. As the cost to cooperate changes (i.e. across MPCR levels) in the one-shot game environment, the estimated distributions of social preference parameters are significantly different. This indicates that cooperative preferences are not stable in one-shot environments as costs change. However, controlling for the cost to cooperate and experience, estimated social preferences are identical across one-shot and repeated games. We do find a significant increase in the error parameter moving from the one-shot to the repeated game environment, and this might help explain why social preferences elicited from one-shot games have not been found to predict choices in repeated games perfectly (Fischbacher & Gächter, 2010). Furthermore, the type of repetition does not have a significant effect on either the distribution of estimated social preferences, the noise parameter or beliefs. This suggests that a possible explanation for the lack of difference in contributions across finitely and indefinitely repeated games is that beliefs and social preferences are identical across both types of repetition.

Our analysis of the role of social preferences in explaining (finitely and indefinitely) repeated public goods games involves an assumption that participants learn beliefs about others’ contributions based on their past choices. We then model behavior as a best response to learned beliefs given social preferences and decision error. While belief learning is one approach to modeling behavior, there may be other ways in which individuals learn. We also consider two competing models of payoff-based individual level learning: a pure reinforcement learning model and a model that mixes reinforcement and belief learning. The simple payoff-based reinforcement learning model fits choices in the repeated game best compared to the alternative models of learning considered.

While the payoff-based reinforcement learning model provides a better fit to the repeated game data, it cannot by itself reconcile behavior across the one-shot and repeated game environments. We use the data from the one-shot game to classify participants into free riders, conditional cooperators and full cooperators. Using these classifications, average contributions across these groups are significantly different in the one-shot and the corresponding repeated game. This indicates that both learning and social preferences differentially matter in explaining contributions in the repeated games. With the help of individual level agent-based simulations, we disentangled the role of social preferences and learning in explaining the repeated game data. We find that a hybrid model that uses social preferences to determine the first round contributions and then payoff-based reinforcement learning in the subsequent rounds generates data that is quantitatively closer to the observed data than a simple reinforcement learning model.

The results in this paper can potentially offer insights into a few robust findings in repeated linear public goods games. The first finding is that when a repeated linear public goods game is restarted, individuals typically start with higher contributions than contributions in the last round of the immediately preceding repeated game (Andreoni, 1988; Croson, 1996). However, the levels of contributions in the first round of the restarted game are smaller than that of the first round contributions in the preceding game. Arifovic and Ledyard (2012) argue that this finding can be explained with a learning model if one
assumes that individuals simply think of the restarted game as a new game and start anew with random contributions and learn over rounds. However, they also note that such an explanation still cannot account for the smaller levels of contributions in the first round of the restarted game. Our experimental results may shed some light on this. The findings are consistent with their explanation that individuals consider the restarted game as a new game and then re-learn over rounds. However, the findings are inconsistent with the assumption that individuals start randomly in the first round of the repeated game. We find that individuals start the repeated game with contributions in line with their social preferences and then use a simple payoff-based reinforcement learning. In addition, our data from the strategy games show that experience has a significant effect on the social preferences of participants by shifting them towards more free riding. The main difference between the first game and the restarted game is the experience of the participants. Our results suggest that first round contributions would be lower in the restart game because there are more free riders.

A second finding is that in experiments with Partners’ and Strangers’ matching designs there is more variability in the contributions in games involving Strangers’ matching (Palfrey & Prisbrey, 1997; Croson, 1996). Andreoni and Croson (2008) conjecture that a possible reason for this could be that Partners’ matching makes learning easier compared to that of Strangers’ matching. The reinforcement learning model we use in this paper can explain high variability in contributions in Strangers matching purely based on learning dynamics. In the reinforcement learning model, the decision error of an individual is proportionally related to the variability of the received payoffs in the previous rounds of the repeated game. In the Strangers’ matching since the group composition changes every round, it is intuitive that received payoffs would be more variable compared to that of the Partners’ matching. The higher variability in received payoffs leads to a higher rate of decision error which translates to an increased variability in contribution choices. Thus, contributions would be more variable in the Strangers’ matching compared to that of Partners’ matching.

Our results also contribute to the ongoing discussion of the relative importance of social preferences and learning in explaining contributions in repeated public goods games. Learning in repeated public goods games is assumed to be individual level directional learning based on social preferences (Anderson, Goeree, & Holt, 2004; Wendel & Oppenheimer, 2010; Cooper & Stockman, 2002; Janssen & Ahn, 2006; Arifovic & Ledyard, 2012). Recent experimental results, however, have found that social preferences may be unnecessary to explain contributions in repeated games and that individuals learn based on payoffs in the game (Burton-Chelley & West, 2013; Burton-Chelley, Nax, & West, 2015). Our analysis shows that the importance of social preferences may be overestimated in the former and underestimated in the latter. Our simulations show that social preferences matter to the extent that they determine the first round contributions, thereafter, individuals’ behavior is explained by payoff-based reinforcement learning.

The paper proceeds as follows. In Section 2 we discuss the experimental design. Section 3 presents a descriptive analysis of the data. Section 4 describes the social preference and learning models, and section 5 outlines the econometric model that is
used in the estimations. In Section 6, we present results. We close with conclusions in Section 7.

2 Experimental Design & Procedures

The experiment uses a linear public goods game. Each individual $i$ is assigned to a group with $N$ members and is given an endowment $w$ that can be invested in a public good account and a private account. The sum of contributions by all group members to the public good account is multiplied by an enhancement factor, $E$, where $1 < E < N$, and the resulting amount is redistributed equally among all the members of the group. Individual $i$’s contribution to the public good account is $c_i \in [0, w]$, and $i$’s payoff is then

$$\pi_i = w - c_i + M \sum_{j=1}^{N} c_j$$

where $M = \frac{E}{N}$ is the marginal per capita return (MPCR) from contributing one unit from the endowment to the public good account. The social dilemma is evident. For any given level of contribution by $i$’s group members, a payoff-maximizing individual’s best response is to contribute nothing. Thus Nash equilibrium predicts a zero contribution from everybody in the group. However, a Pareto optimum is achieved when everybody contributes the entire endowment to the public good. In all decisions, there are three individuals in each group, and the endowment is 20 tokens. All decisions were made with tokens, and converted into a monetary payoff at a rate of 20 tokens = $1.

The experiment is based on the Fischbacher and Gächter (2010) design. Each participant is asked to complete four tasks in the following order: P1, R1, P2, R2. In the $P$-task, the participant completes a conditional contribution table in which he decides how much to contribute to the public good account for 21 possible average contribution amounts of his group members (e.g. 0, 1, 2, ..., 20). He also makes an unconditional contribution decision by choosing how many of the 20 tokens to contribute to the public good. Payoffs from this task are determined as follows. Once all decisions are completed by the group members, two group members are chosen at random. The unconditional contributions of those two group members are averaged together and rounded up or down to the nearest integer $k$. The contribution of the third group member is determined by the amount specified in the conditional contribution table for the average contribution amount of $k$ by the other two group members. The total amount then contributed to the public good account is the sum of the unconditional contributions of the first two group members and the conditional contribution of the third. Participant payoffs are based on this. Everyone knows these procedures before making their decisions.

In the $R$-task, participants make an unconditional decision of how many tokens to put in the public good account. This decision is repeated over several rounds, and the members of a group are fixed for all rounds in an R-task. This is a partners matching protocol. After deciding how much to contribute to the public good account, a participant is asked to state his belief of the average contribution of the other two group members in
the current round. Participants were paid for the accuracy of this stated belief. At the end of a round, a participant is informed of the exact contribution of each group member, the average contribution, and his payoff for that round.

In sessions with finitely repeated games, the number of rounds was fixed at seven. In the sessions with indefinitely repeated games, there was at least one round and then after that the probability of a subsequent round was 0.85. The continuation probability of 0.85 yields, on average, seven rounds of play so the finitely repeated games was fixed at seven rounds to make the games comparable in the expected number of rounds.

Each session had exactly 15 participants and thus 5 groups in each of the tasks. Participants were randomly reshuffled across groups before each of the tasks and subjects were aware of this before making decisions. Tasks P1 and R1 use the same MPCR level, and tasks P2 and R2 also use the same MPCR level. The MPCR used for P1 and R1 is different than that used for P2 and R2. The two MPCR levels are 0.4 (Low) and 0.8 (High). In five sessions, the low MPCR was used for P1 and R1 and the high MPCR was used for P2 and R2. To control for order effects, in five sessions, this was reversed. Each participant completed all four tasks. Participants knew that there would be four tasks, and the instructions for each task were distributed and read out loud prior to the start of each task. To make sure participants understood the decisions they are asked to make and how to calculate payoffs, a short quiz was administered prior to the start of Task P1. Answers to the quiz were explained before proceeding to the experiment. Participants were paid their earnings for all tasks and all rounds within a task. Earnings were paid privately and in cash at the end of the session.

Participants made their decision on a computer, using a web-based software. The experiments were run in the Interdisciplinary Center for Economic Science (ICES) at George Mason University during September and October of 2014. Ten sessions were run, and there were a total of 150 participants. Participants were randomly assigned to cubicles and made their decisions privately and anonymously. Six of the ten sessions involved indefinitely repeated games and four sessions involved finitely repeated games. No one participated in more than one session. Participants were recruited via email from a pool of George Mason University students who had all previously registered to receive invitations for experiments. Each experimental session lasted for approximately 1.5 hours. Average participant earnings were $26.36 (s.d. $8.67).
Table 1: Types in Strategy Games (using P-task data)

<table>
<thead>
<tr>
<th>Type</th>
<th>P1</th>
<th>P2</th>
<th>Low MPCR (0.4)</th>
<th>High MPCR (0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Riders</td>
<td>36 (24%)</td>
<td>54 (36%)</td>
<td>54 (36%)</td>
<td>36 (24%)</td>
</tr>
<tr>
<td>Conditional Coop.</td>
<td>89 (59%)</td>
<td>69 (46%)</td>
<td>71 (47%)</td>
<td>87 (58%)</td>
</tr>
<tr>
<td>Full Contributors</td>
<td>17 (11%)</td>
<td>20 (13%)</td>
<td>18 (12%)</td>
<td>19 (13%)</td>
</tr>
<tr>
<td>Noisy Contributors</td>
<td>8 (5%)</td>
<td>7 (5%)</td>
<td>7 (5%)</td>
<td>8 (5%)</td>
</tr>
<tr>
<td>Total</td>
<td>150 (100%)</td>
<td>150 (100%)</td>
<td>150(100%)</td>
<td>150 (100%)</td>
</tr>
</tbody>
</table>

3 Data

In this section, we describe the conditional contribution decisions from the P-tasks and the contribution and belief decisions from the R-tasks.

To examine the conditional contributions in the P-tasks, we classified participants into types using the statistical classification algorithm of Kurzban and Houser (2005). This algorithm uses a linear conditional-contribution profile (LCP) to determine a given participant’s type. The LCP is the result of an ordinary least squares regression of a participant’s conditional contribution in the P-task on each of the 21 possible average contributions of the other group members. If the estimated LCP is strictly below half the endowment everywhere, then the participant is classified as a Free Rider (FR). A participant is classified as a Full Cooperator (FC) if the LCP lies at or above half the endowment everywhere. If the LCP of a participant has a positive slope and lies both above and below half the endowment then he is a Conditional Cooperator (CC). Any participant who did not fall into one of these three categories is classified as a Noisy Contributors (NC).

The distribution of types identified with the classification algorithm is presented in Table 1. The first two columns present the distributions observed in the P1 and P2 tasks. The third and fourth columns present the distributions observed in the low MPCR and high MPCR P-tasks respectively. Consistent with previous studies, most participants are classified as conditional cooperators, and roughly one quarter are classified as free riders. These two types account for more than 80% of participants in the combined P1 and P2 tasks and also when separated by the high and low MPCR treatments. Full cooperators and noisy contributors are less frequent. From the P1 to P2 task, as experience increases, there is a significant change in the distribution of types, with an increase in the proportion of free riders and a decrease in the proportion of conditional cooperators (Chi-Squared Test, $\chi^2(3) = 12.39; p = 0.006$). From the low to high MPCR, as the cost to cooperate declines, there is also a significant shift in the distribution, with the proportion of full cooperators rising (Chi-Squared Test, $\chi^2(3) = 12.12; p = 0.007$).

Looking at the contribution behavior of each type, in P1, the average contribution

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5 In Kurzban and Houser (2005), the authors classify individuals into only the first three groups and exclude analysis on three participants who did not fall into these categories.

6 Previous studies include Fischbacher, Gächter, and Fehr (2001); Fischbacher and Gächter (2010); Kurzban and Houser (2001, 2005); Burlando and Guala (2005); Duffy and Ochs (2009); Kocher, Cherry, Kroll, Netzer, and Sutter (2008); Herrmann and Thöni (2009); Muller, Sefton, Steinberg, and Vesterlund (2008); Bardsley and Moffatt (2007)
across all conditional contributions is 2.39 tokens for free riders, 9.66 for conditional cooperators, 18.10 for full contributors and 10.45 for noisy contributors. In P2, the average contribution is 2.47 for free riders, 9.69 for conditional cooperators, 17.53 for full contributors, and 9.80 for noisy contributors. In both P1 and P2, average contributions across types are significantly different (Kruskal-Wallis tests, \( p < 0.0001 \)).

Turning now to the R-task, four of our ten experimental sessions involved finitely repeated games of seven rounds each. The remaining six sessions used indefinitely repeated games. The probabilistic continuation rule produced rounds of the following length for R1 and R2 across the six sessions: \{ {6,7}, \{3,25\}, \{7,12\}, \{5,2\}, \{9,5\}, \{6,5\} \}. While the finite and indefinitely repeated games had different lengths, we find no significant difference in average contributions across these games.\(^7\) The high MPCR results in higher contributions in both the finitely and indefinitely repeated games, and this is consistent with previous research (Isaac, Walker, & Thomas, 1984; Lugovskyy et al., 2015).\(^8\) We also find that beliefs are identical across finitely and indefinitely repeated games.\(^9\) The round level contributions and beliefs are positively and significantly correlated in all ten sessions, as also found in previous studies (Fischbacher & Gächter, 2010; Croson, 2007; Weimann, 1994).\(^10\) Appendix 2 provide plots of average contributions and beliefs across groups.

## 4 Social Preferences and Learning Models

In this section, we describe the utility specification and learning models that form the basis of our econometric analysis of social preferences and learning. We use the utility function of Arifovic and Ledyard (2012) in the basic specification because it models social preferences that include altruism and a concern with one-sided fairness, as well as accommodating changes in the cost to cooperate. We then proceed to a description of the learning models that we use to explain the repeated game data.

### 4.1 Arifovic-Ledyard model of social preferences

We use the linear utility model with social preferences as developed in Arifovic and Ledyard (2012) for the context of linear public goods games. Consider a group of size \( N \) and an MPCR level \( M \). Each individual \( i \in \{1, 2, ..., N\} \) is endowed with \( w \). The payoff

\( ^7\)This was also found by Lugovskyy et al. (2015). We test for the effect of indefinite repetition by clustering at the session level and find no effect on contributions (\( F(1,9) = 0.24, p = 0.634 \)). Identical conclusions are reached using clustering at the participant level. Since the participants are shuffled across 3-person groups at the start of each of the strategy and repeated games, we favor the session level clustering to report our results.

\( ^8\)All sessions: \( F(1,9) = 16.78, p = 0.0027 \); Finitely Repeated: \( F(1,3) = 14.96, p = 0.0306 \); Indefinitely Repeated: \( F(1,5) = 13.31, p = 0.0148 \).

\( ^9\)We test for the effect of indefinite repetition by clustering at the session level and find no effect on beliefs (\( F(1,9) = 0.37, p = 0.557 \)).

\( ^10\)The correlations in each of the ten sessions are: 0.61, 0.81, 0.73, 0.50, 0.76, 0.82, 0.41, 0.62, 0.62, and 0.72. Spearman rank correlation tests, \( p < 0.0001 \).
an individual $i$ receives by contributing $c^i$ when others in his group contribute on average $o$ can be written as $\pi^i(c^i, o) = w - c^i + M(c^i + (N - 1)o)$. Similarly, the average payoff of the group can be written as $\bar{\pi}(c^i, o) = w - \bar{c} + MN\bar{c}$, where $\bar{c} = \frac{c^i + (N-1)o}{N}$. The utility derived by the individual $i$:

$$u^i(c^i, o) = \pi^i(c^i, o) + \beta^i \bar{\pi}(c^i, o) - \gamma^i \max\{0, \bar{\pi}(c^i, o) - \pi^i(c^i, o)\}$$

Where $\beta^i \geq 0; \gamma^i \geq 0$ are social preference parameters. $\beta^i > 0$ implies that individual $i$ has a preference for a higher average payoff of his group members. In other words, $\beta^i$ characterizes an individual’s altruistic preference. $\gamma^i > 0$ implies that individual $i$ obtains a disutility when his payoff is smaller than the average payoff of the group, i.e. when $\bar{\pi}(c, o) > \pi^i(c, o)$. $\gamma^i$ captures the disutility individual $i$ faces when his payoff is lower than the group average.

In the equilibrium individual $i$ would choose a contribution $c^i$ as follows:

$$c^i = \begin{cases} 
0 & \text{if } 0 \geq \left( M - \frac{1}{N} \right) \beta^i + M - 1 \\
\bar{c} & \text{if } \gamma^i \left( \frac{N-1}{N} \right) \geq \left( M - \frac{1}{N} \right) \beta^i + M - 1 \geq 0 \\
w & \text{if } \gamma^i \left( \frac{N-1}{N} \right) \leq \left( M - \frac{1}{N} \right) \beta^i + M - 1
\end{cases}$$

The social preference parameters ($\beta^i, \gamma^i$), along with the parameters of the public goods game ($N, M$), determine if individual $i$ behaves as a free rider, a conditional co-operaor or a pure altruist (full cooperator). For example, consider a participant with $\beta = 2$ and $\gamma = 1$, with $N = 3$ and $M \in \{0.4, 0.8\}$ as in our experiments. In equilibrium, this participant will give nothing when $M = 0.4$, acting as a free rider, however, when $M = 0.8$ he will contribute all of the endowment, acting as a full cooperater. Thus, the model provides a theoretical foundation of preferences that can explain observed changes in conditional contribution decisions and permits testable predictions of the distribution of social preferences as MPCR’s change.

The theory does not allow all possible type switches when MPCR changes. Appendix 3 presents the ranges of permissible $\beta$ and $\gamma$ for behavioral types at different levels of MPCR in our experiment. It also presents which type switches are possible and which are not.

### 4.2 Learning models

We consider three models to understand how participants learn in the repeated games. A brief description of each of the learning models is presented below.

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11While social preference models of Fehr and Schmidt (1999) and Charness and Rabin (2002) also capture the effect of MPCR (M) on the behavioral type of an individual, they suggest behavior is independent of group size. Thus, they do not pick up the variation in observed contributions across small and larger groups as identified in Isaac, Walker, and Williams (1994); Isaac and Walker (1988). The other regarding preference model of Arifovic and Ledyard (2012) accounts for this variation.
4.2.1 Social preferences based belief learning model

In this model we assume that the participants are stochastically best responding to their stated beliefs according to the utility function in (1). Thus learning is captured by updating stated beliefs about the average contribution of the other group members over time in the repeated game. Using stated beliefs makes the estimations simpler by removing an added layer of econometric modeling of belief updating. Nonetheless, we do compare the performance of stated beliefs that are learned by some cognitive mechanism and estimated beliefs from the empirical model of belief learning by Cheung and Friedman (1998) in the prediction section.

4.2.2 Payoff-based reinforcement learning (REL)

The idea of reinforcing actions that led to higher payoffs in the previous rounds of play with higher probabilities is one of the simplest and oldest of learning models. We consider a variation of a reinforcement learning model known as the REL model introduced by Erev, Berely-Meyer, and Roth (1999).

In the REL model, contribution decisions in round \( t \) are made based on the “attraction” \( A_j(t) \) of each contribution level \( j \). Let a contribution \( j \) of an individual \( i \) have an attraction \( A_{ji}^t(t) \) in a round \( t \). The probability that individual \( i \) chooses the strategy \( j \) in round \( t \) is given by:

\[
P_{ji}^t(t) = \frac{e^{\lambda_i \cdot PV_{ji}^t(t)}}{\sum_{l=0}^{20} e^{\lambda_i \cdot PV_{ji}^t(t)}}
\]

where \( \lambda_i \) is a free parameter that determines the reinforcement sensitivity of the individual \( i \). \( PV_{ji}(t) \) is the measure of payoff variability.

If the contribution \( j \) was chosen in a round \( t \) and other individuals in \( i \)’s group contributed on average \( o(t) \) in that round, then \( i \) would receive a payoff of \( \pi^i(j,o(t)) \). The attraction of the strategy \( j \) of the individual \( i \) is then updated as below:

\[
A_{ji}^t(t+1) = \left[ \frac{A_{ji}^t(t)C_{ji}^t(t) + N_i(1)}{C_{ji}^t(t) + N_i(1) + 1} + \pi^i(j,o(t)) \right]
\]

where \( C_{ji}^t(t) \) is the number of times that contribution \( j \) has been chosen in the first \( t \) rounds and \( N_i(1) \) is a free parameter that determines the strength of the initial attractions. The attractions of unchosen strategies are not updated. Attractions in the first round, \( A_{ji}^t(1) \), \( \forall j \), are initialized using the first round contributions (see Appendix 4).

The payoff variability is updated according to:

\[
PV_{ji}(t+1) = \frac{[PV_{ji}(t) + mN_i(t)] + |\pi^i(j,o(t)) - PA_i(t)|}{t + mN_i(t) + 1}
\]

where \( PA_i(t) \) is the accumulated payoff average in round \( t \) and \( m \) is the number of strategies (21 in the current context since there are 21 levels of contributions possible).
$PV_i(1) > 0$ can be either estimated as a free parameter or set to a constant, for example, to the expected absolute difference between the obtained payoff from a random choice and the average payoff from a random choice as done in Erev et al. (1999).\textsuperscript{12}

$PA_i(t)$ is calculated in a similar manner. We initialized $PA_i(1) = 0$ since there was no payoff before the first round\textsuperscript{13}, and

$$PA_i(t + 1) = \frac{[PA_i(t)(t + mN_i(1)) + \pi^i(j, o(t))]}{[t + mN_i(1) + 1]}$$

There are two free parameters to be estimated in the REL model: the noise parameter $\lambda_i$ and the strength of initial attractions $N_i(1)$.

4.2.3 Payoff-based self-tuning EWA

The Self-Tuning Experience Weighted Attraction (STEWA) learning model was introduced by Ho, Camerer, and Chong (2007) as a one-parameter variation of the original Experience Weighted Attraction (EWA) model of C. Camerer and Ho (1999). STEWA fixes the parameters \{\kappa, N(0)\} of the EWA model at $\kappa = 0$, $N(0) = 1$. It also replaces the two other parameters $\phi$ and $\delta$ of the EWA model with functionals $\phi(t)$ and $\delta(t)$ respectively.

As in the REL model, in STEWA, contribution decisions in round $t$ are made based on the “attraction” $A_j(t)$ of each contribution level $j$. Let a contribution $j$ of an individual $i$ has an attraction $A_j^i(t)$ in a round $t$, then the probability that the individual $i$ chooses the strategy $j$ in the round $t$ is given by:

$$P_j^i(t) = \frac{e^{\lambda_i A_j^i(t)}}{\sum_{l=1}^{m} e^{\lambda_l A_l^i(t)}}$$

where $\lambda_i$ is a free parameter that determines the level of noise in decision making of the individual $i$. If the contribution $j$ was chosen in a round $t$ and other individuals in $i$’s group contributed on average $o(t)$ in round $t$, then $i$ would receive a payoff of $\pi^i(j, o(t))$ in that round. The attraction of the strategy $j$ of the individual $i$ is then updated as below:

Attraction of each contribution $q$ in the round $t + 1$ is determined as:

$$A_j^q(t + 1) = \phi_i(t + 1).N(t).A_j^q(t) + [\delta_iq(t + 1) + (1 - \delta_iq(t + 1))].\mathbb{I}(j, q)\.\pi_i(q, o(t))$$

Where $\mathbb{I}(q, j)$ is an Indicator function and equals 1 when $q = j$. Otherwise it is zero.

Each attraction is applied an experience weight using:

\textsuperscript{12}Our preliminary estimations showed that estimating $PV_i(1)$ made a little difference in the fit and we fixed it at 1

\textsuperscript{13}Alternatively we could initialize $PA_i(1)$ as equal to the average payoff of a random choice like Erev et al. (1999) did or estimate it. It did not make much difference to the fit in either case.
\[ \text{N}(t + 1) = 1 + \text{N}(t)\phi(t + 1), \text{ with } \text{N}(0) = 1 \]

The change-detector function \( \phi(t+1) \) weights the lagged attractions and characterizes the individual’s perception about how quickly the learning environment is changing (Ho et al., 2007). It is defined as,

\[ \phi_i(t + 1) = 1 - \frac{1}{2}S_i(t + 1); \]

\[ S_i(t + 1) = 20 \sum_{k=0}^{20} (h^k_i(t + 1) - r^k_i(t + 1))^2 \]

where \( k \) is the average contribution of others. The cumulative history vector \( h^k_i(t + 1) \) records the historical frequencies of average contributions \( k \)'s of other group members (including the last period \( t \)).

\[ h^k_i(t + 1) = \sum_{\tau=1}^{t} \mathbb{I}(k, o(\tau)) \]

where \( o(\tau) \) is the observed average contribution of others in the round \( \tau \). The immediate history vector \( r^k_i(t + 1) \), is a vector of 1’s and 0’s. Therefore in the round \( t + 1 \), \( r^k_i(t + 1) = \mathbb{I}(k, o(t)) \). \( S(t+1) \) is the quadratic distance between the cumulative history vector \( h^k_i(t + 1) \) and the immediate history vector \( r^k_i(t + 1) \). It captures the degree of surprise due to changes in the observed contributions of other group members. It will always lie between 0 and 2 and so \( \phi_i(t + 1) \) lies in between 0 and 1.

The last component of the STEWA model, the attention function \( \delta_i \), generates a weight for foregone payoffs and turns the attention to the strategies that would have led to higher payoffs. \( \delta_i^q(t + 1) \) is given as:

\[ \delta_{iq}(t + 1) = \begin{cases} 
1 & \text{if } \pi_i(q, o(t)) \geq \pi_i(j, o(t)) \\
0 & \text{Otherwise} 
\end{cases} \]

5 Econometric Framework

In this section, we formulate structural econometric models of discrete choice that can be estimated by maximum likelihood to estimate the social preference parameters and learning parameters allowing for heterogeneity. This is the appropriate approach when using the data generated in our experiments because choices are made on a discrete scale.

5.1 Estimation of social preferences from the strategy games

Using the specification of the utility function that includes social preferences in Equation 1, let \( T_i \) be the number of decision situations a participant \( i \) has faced in the strategy games. Let \( C_i = \{c_{it}| t \in \{1, 2, ...., T_i\}\} \) be the vector of observed contributions of the participant \( i \) and \( O_i = \{o_{it}| t \in \{1, 2, ...., T_i\}\} \) be the vector of the average contributions.
of i’s group members (excluding i himself). The average contributions of other group members are stated explicitly in the P-tasks.

We begin developing our econometric model by assuming the participants’ decisions reflect maximization of a utility function with social preferences as specified in Equation 1. In the absence of any errors in decision making, for a given level of average contribution of others in the group, a participant chooses a contribution that maximizes his utility.

As a first step to allow for stochastic decision making, we add a standard extreme value distributed error term to the utility derived from each level of contribution. Assuming that these errors are independent of other parameters of the model and regressors, we obtain the logit probability of choosing a contribution level $c_{it}$ as:

$$l_{it} = \frac{e^{U_{it}(c_{it}, o_{it})}}{\sum_{j=0}^{20} e^{U_{it}(j, o_{it})}}$$

The errors from a standard extreme value distribution that are added to the utility capture the idea that a participant’s computation of his utility may be subject to some variability (Loomes, 2005). A number of experimental studies involving public goods games have found evidence for “trembles” or “random choice errors” (Bardsley & Moffatt, 2007; Moffatt & Peters, 2001). These “trembles” account for a participant’s failure to understand the decision problem or attention lapses during decision making. We model the propensity of a participant to choose randomly in any given task with a “trembling hand” parameter called $\omega_i$.

Since each participant is endowed with 20 tokens, the probability of choosing a given contribution level via random choice is $\frac{1}{21}$.

Then, for the participant $i$, the probability of an observed level of contribution $c_{it}$ for an average contribution $o_{it}$ of his group members can be written as:

$$l_{it}(c_{it}, o_{it}, \beta_i, \gamma_i, \omega_i) = (1 - \omega_i) \frac{e^{U_{it}(c_{it}, o_{it})}}{\sum_{j=0}^{20} e^{U_{it}(j, o_{it})}} + \frac{\omega_i}{21}$$

A random coefficient model is used to estimate the distribution of the individual-specific structural parameters $\beta_i, \gamma_i$ and $\omega_i$ in the population. This approach is favored over separate estimations for each individual since the number of observed choices per individual is rather small across the four tasks.

Since $\beta, \gamma$ are constrained to be positive we modeled them using log-normal distributions. To bound $\omega$ between 0 and 1, we modeled it as a logistic-normal distribution.
over \([0, 1]\). For a concise notation define,

\[ \eta_i = g_{\eta_i}(X^\eta_i\delta^\eta + \xi^\eta_i), \eta_i \in \{\beta_i, \gamma_i, \omega_i\} \]

\( \eta_i \) denotes one of the three individual specific parameters, \( X^\eta_i \) are \( 1 \times K^\eta \) vectors of regressors, \( \delta^\eta \) are \( K^\eta \times 1 \) parameter vectors, and \( \xi^\eta_i \) are the unobserved heterogeneity components of the parameters. The first element of each \( X^\eta_i \) contains 1. The functions \( g_{\eta}(.) \) impose theoretical restrictions on the individual specific parameters. For \( \beta, \gamma \), the exponential function ensures that they are positive. For \( \omega \), the logistic distribution function ensures that it is always between zero and one. \( g(X, \delta + \xi_i) \) stands for a vector of the three functions.

We assume that \( \xi_i = (\xi^\beta_i, \xi^\gamma_i, \xi^\omega_i)' \) follows a jointly normal distribution with a diagonal covariance matrix \( \Sigma \) independent of the regressors. The regressor matrix contains only ones in the minimal estimation. In the full estimation case, it contains a dummy for the high MPCR, a dummy for the repeated game, and a dummy for experience (which is one for the second strategy game (P2) and the second repeated game (R2)).

The likelihood contribution of participant \( i \) can be written as:

\[ l_{it} = \int_{\mathbb{R}^3} \left[ \prod_{t=1}^{T_i} l_{it}(c_{it}, o_{it}, g(X_i\delta + \xi)) \right] \phi(\xi) d\xi \] (4)

where \( l_{it} \) is the probability given in Equation 3 and \( \phi(.) \) denotes the density of multivariate normal \( \xi \). The above integral does not have a closed form solution. We approximate it using \( R = 1000 \) Halton draws from \( \xi \) to obtain a simulated likelihood (Train, 2009; Bhat, 2001). The simulated likelihood contribution of participant \( i \) is:

\[ sl_i = \sum_{r=1}^{R} \frac{l_i(\xi_r)}{R} \]

The (simulated) log-likelihood is given by the sum of the logarithms of \( sl_i \) over all participants in the experiment. We maximized the log-likelihood function of all the participants using a two-step hybrid approach and repeated a multiple number of times as discussed in Liu and Mahmassani (2000) to avoid local maxima\(^\text{17} \). The variance-covariance matrix of the parameter estimates is computed using the sandwich estimator (Wooldridge, 2010). Standard errors are calculated using the sandwich estimator and treating all choices of each participant as a single super-observation, that is, using degrees of freedom equal to the number of participants rather than the number of participants.

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\(^\text{17} \)In the first step, we have employed a genetic algorithm to find the parameters that maximize the log-likelihood of the sample. Genetic algorithms are very effective in searching many peaks of the likelihood function based on a rich “population” of solutions and thus reduce the probability of getting trapped at a local maximum. Since they do not require gradients to be computed, they are computationally very efficient for a global search of the parameters. In the second step, we used the solution of the genetic algorithm as a starting point to Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm with numerical derivatives to maximize the log-likelihood function.
times the number of choices made. Standard errors for transformed parameters are calculated using the delta method.

5.2 Estimation of learning models

Estimation of the belief learning model with social preferences is straightforward. In R-tasks, a belief about the average contribution of other group members is elicited from the participants. The belief learning model assumes that the participants stochastically best respond (subject to noise in decision making) to their beliefs in each round. The estimation methodology is then identical to the estimation of the random coefficients model used to estimate the structural parameters described in the context of the strategy game data.

The REL model has two structural parameters: a noise parameter $\lambda_i$ and a parameter that defines the initial strength of attractions $N_i(1)$. We model both of them as random coefficients allowing for individual level heterogeneity.\(^{18}\) Using the notation in the previous subsection:

$$\eta_i = g_\eta(X_\eta_i \delta_\eta + \xi_\eta_i), \eta_i \in \{\lambda_i, N_i(1)\}$$

$\eta_i$ denotes one of the two individual specific parameters, $X_\eta_i$ are $1 \times K^\eta$ vectors of regressors, $\delta_\eta$ are $K^\eta \times 1$ parameter vectors, and $\xi_\eta_i$ are the unobserved heterogeneity components of the parameters. The first element of each $X_\eta_i$ contains 1. Since both $\lambda_i$, $N_i(1)$ are positive, we used the exponential function for $g_\eta$ for both parameters. Assuming that $\xi_i = (\xi_i^{\lambda}, \xi_i^{N(1)})'$ follows a jointly normal distribution with a diagonal covariance matrix $\Sigma$ independent of the regressors, the likelihood contribution of participant $i$ can be written as:

$$l_i = \int_{\mathbb{R}^2} \left[ \prod_{t=1}^{T_i} \left( \sum_{l=0}^{20} \mathbb{I}(c_{it}, l) P_l^{\eta}(t) \right) \right] \phi(\xi) d\xi$$

where

$$P_l^{\eta}(t) = \frac{e^{\lambda_i P_{Vi}^{\eta}(t)}}{\sum_{k=0}^{20} e^{\lambda_i P_{Vi}^{\eta}(t)}}$$

is the probability of choosing contribution level $l$ in round $t$. $c_{it}$ is the observed contribution in $t$. $\mathbb{I}(c_{it}, l) = 1$ if $c_{it} = l$, 0 otherwise. $T_i$ is the total number of rounds in the repeated games R1 and R2 that individual $i$ has participated in (it should be noted that attractions, payoff variability, and accumulated payoff average will be reinitialized at the start of the R2). The integral in Equation 5 is computed using simulations and the total log-likelihood of the sample is computed as the sum of the logarithms of the simulated individual level likelihoods of all participants. We maximized the log-likelihood function of all the participants using a two-step hybrid approach (described in the previous subsection) and repeated a multiple number of times to avoid local maxima. The

\(^{18}\)An individual level random choice error $\omega_i$ can also be estimated along with $\lambda_i$ in the learning models, however our preliminary estimations showed that it does not increase the fit significantly.
variance-covariance matrix of the parameter estimates is computed using the sandwich estimator.

The STEWA model has one structural parameter \( \lambda_i \in \mathbb{R}^+ \), which characterizes the noise in the decision making of a participant. We model it as a random coefficient and estimate it using the method of simulated maximum likelihood as described in the context of the REL model.

6 Results

We present our results in stages. First, using the structural econometric approach outlined in the previous section, we examine how estimated social preferences from decisions in a one-shot environment, using data from the P-tasks, are affected by the cost to cooperate and experience. Second, we ask if estimated social preferences differ across one-shot and repeated environments by using the data from the P-tasks and the R-tasks. Third, using the repeated game data we explore whether the type of repetition, finite or indefinite, has a significant effect on estimated social preferences. Fourth, we look at how well individual level estimates of social preferences in one-shot environments can explain choices in the repeated games. Fifth, we examine choices solely in the repeated game environment by comparing a number of representative learning models. We conclude with a simulation study in which we assess the roles of social preferences and learning in explaining choices in the repeated game environment.

6.1 Stability of social preferences in the one-shot environment

Choices in the P-task provide data to directly test the effects of changes in the MPCR on the distribution of underlying social preferences. The decisions made in this task do not require computations of beliefs about others’ contributions since others’ contributions are explicitly presented. Choices should then reflect best responses given social preferences as accurately as possible. Of course, decisions errors are possible, and our estimation approach takes this into account.

Table 2 reports the estimates for the three parameter vector \( \delta^\eta, \eta \in \{\beta, \gamma, \omega\} \) for the model that contains an intercept and treatment dummies for experience and high MPCR. The values in the table are in the original parameters’ scale. The constant term is \( g_\eta(\delta^\eta_1) \) and represent the median parameter value for the baseline treatment, which is the P1 task with a low MPCR. Treatment effects reported in the table are the calculated partial effects of moving from the baseline treatment to the treatment under consideration using the median parameter value. For example, the value shown in the table for the high MPCR dummy is \( g_\eta(\delta^\eta_1 + \delta^\eta_{\text{High MPCR}}) - g_\eta(\delta^\eta_1) \) and represents the partial effect of moving from the baseline to the high MPCR treatment on the parameter \( \eta \in \{\beta, \gamma, \omega\} \).

For illustration, in the estimations we have, \( \delta^\beta_1 = 3.44 \) and \( \delta^\beta_{\text{High MPCR}} = -1.75 \). Note that \( g_\beta \) is the exponential function. The corresponding treatment effect of the high MPCR on the median value of \( \beta \) is computed as \( e^{(3.44 - 1.75)} - e^{3.44} = -25.89 \) and is reported in the table.

For illustration, in the estimations we have, \( \delta^\beta_1 = 3.44 \) and \( \delta^\beta_{\text{High MPCR}} = -1.75 \).Note that \( g_\beta \) is the exponential function. The corresponding treatment effect of the high MPCR on the median value of \( \beta \) is computed as \( e^{(3.44 - 1.75)} - e^{3.44} = -25.89 \) and is reported in the table.
Table 2: Treatment effects on $\beta$, $\gamma$, and $\omega$ in strategy games (P-tasks)

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>31.30***</td>
<td>8.89***</td>
<td>0.0042</td>
</tr>
<tr>
<td></td>
<td>(3.62)</td>
<td>(0.79)</td>
<td>(0.0048)</td>
</tr>
<tr>
<td>Experience</td>
<td>-22.21***</td>
<td>0.11</td>
<td>-0.0042</td>
</tr>
<tr>
<td></td>
<td>(2.75)</td>
<td>(1.20)</td>
<td>(0.0048)</td>
</tr>
<tr>
<td>High MPCR</td>
<td>-25.89***</td>
<td>-6.26***</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(3.15)</td>
<td>(0.76)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.21***</td>
<td>2.06***</td>
<td>13.99***</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.050)</td>
<td>(2.13)</td>
</tr>
</tbody>
</table>

$N$ 150
$LL$ -13625

Notes: *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$. Coefficients are transformed back to the original scale. A treatment effect is the (partial) effect of moving from the baseline treatment to the corresponding treatment on the median parameter value. Standard errors are calculated using the sandwich estimator and treating all of each subject’s choices as a single super observation. The entries for $\sigma$ are the standard deviations of the untransformed normal distributions of the random coefficients.

In the P1-task with a low MPCR, the median values for $\beta, \gamma, \omega$ are 31.30, 8.89, 0.0042. Using the categorization outlined in Appendix 3, the median participant in the baseline treatment acts as a conditional cooperator. This is in line with the classification results using the LCP method presented in Appendix 8 (see column P1-Low). In terms of decision error, the median random choice probability ($\omega$) is 0.42% in the baseline treatment (P1-Low). Less than 0.5% of the choices can be characterized as being made randomly in the baseline treatment, and there are no significant effects of the MPCR or experience on decision error. This could be due to the strategic simplicity of the strategy game where choices are conditional on the average contributions of others.

To understand our estimates of treatment effects on the social preference parameters, it is important to keep in mind that the distribution of types (free riding, conditional cooperation, full cooperation) are determined by the joint distribution of $\beta, \gamma$ conditional on the group size ($N$) and the MPCR ($M$).\textsuperscript{20} Figure 1 shows the proportions of types by MPCR and experience using estimates in Table 2.\textsuperscript{21} The figure shows that the proportion of noisy contributors declines with experience, moving from 7% in P1-low to 0% in P2-low and 28% in P1-high to 0% in P2-high. Also, the proportion of full contributors rises with an increase in MPCR level, moving from 25% in P1-Low to 40% in P1-High and from 10% in P2-Low to 29% in P2-High. This is consistent with changes in the distribution of types based on the LCP classification algorithm reported in Table 1. Furthermore, the proportion of free riders substantially rises with experience, moving from 18% in P1-Low

\textsuperscript{20}The social preference parameter ranges that lead to different behaviors are reported in Appendix 3.

\textsuperscript{21}Here, we drew a sample of size 100,000 from the joint distribution of $\beta, \gamma, \omega$ using the estimates. First, we identified the individuals whose $\omega$ is larger than 0.5 as being noisy contributors. For the remaining, we used the calculations in Appendix 3 to determine their behavioral types in a given treatment.
to 48% in P2-Low and from 3% in P1-High to 19% in P2-High. Thus we observe an increase in selfish behavior with experience similar to the findings of Brosig et al. (2007) and Sass and Weimann (2012).

Using the P-task choices, the estimates show that the observed change in contribution behavior is a reflection of a shift in underlying social preferences for altruism and fairness as the cost to cooperate change and individuals gain experience. That is, we do not find support for the stability of social preferences in the one-shot games.

6.2 Transferability of social preferences across one-shot and repeated environments

Next, we econometrically test if the type of the game, strategy or repeated, influences preferences for altruism and fairness. To do so, we use data from both the P-tasks and the R-tasks. The estimation assumes that participants are stochastically best responding to their stated beliefs in the repeated games.

Table 3 contains the estimates for the three parameter vector $\delta^\eta, \eta \in \{\beta, \gamma, \omega\}$ for the model that contains an intercept and treatment dummies for experience, high MPCR and whether or not the data is from the repeated game. All parameters are on the original scale. In the baseline treatment, median values for $\beta, \gamma, \omega$ are 39.01, 16.08, 0.051. The effects of the MPCR and experience are similar to those found using only the P-task decisions. Using the computations in Appendix 3, as was found with the estimates from Table 2, the median participant acts as a conditional cooperator in the baseline treatment.

There is no significant effect of the repeated game dummy on the social preference parameters. However, there is a significant and substantial increase in the noise parameter in the repeated games. There are two potential explanations for higher decision error. First, contribution decisions in repeated games are more complex than conditional
Table 3: Treatment effects on $\beta$, $\gamma$, and $\omega$ in combined dataset (P- and R-tasks)

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>39.01***</td>
<td>16.08***</td>
<td>0.051</td>
</tr>
<tr>
<td></td>
<td>(1.13)</td>
<td>(0.30)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>Experience</td>
<td>-16.18***</td>
<td>-13.80***</td>
<td>-0.031</td>
</tr>
<tr>
<td></td>
<td>(3.67)</td>
<td>(0.49)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>High MPCR</td>
<td>-35.03***</td>
<td>-12.64***</td>
<td>-0.038</td>
</tr>
<tr>
<td></td>
<td>(1.13)</td>
<td>(0.49)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Repeated</td>
<td>15.43</td>
<td>-2.55</td>
<td>0.58**</td>
</tr>
<tr>
<td></td>
<td>(18.26)</td>
<td>(4.62)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.68***</td>
<td>1.62***</td>
<td>3.20***</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.01)</td>
<td>(0.67)</td>
</tr>
<tr>
<td>N</td>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>-18400</td>
<td></td>
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</tr>
</tbody>
</table>

Notes: *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$. Coefficients are transformed back to the original scale. A treatment effect is the (partial) effect of moving from the baseline treatment to the corresponding treatment on the median parameter value. Standard errors are calculated using the sandwich estimator and treating all of each subjects choices as a single super observation. The entries for $\sigma$ are the standard deviations of the untransformed normal distributions of the random coefficients.

It is noteworthy that the only significant effect of the repeated game is through the random choice parameter rather than via the social preference parameters. Controlling for the MPCR and experience, the strategic environment does not have significant effects on the distribution of social preferences. In other words, social preferences are transferable across strategy and repeated decisions.

6.3 Effect of finite and indefinite repetition on social preferences

We would like to know if the type of repetition, finite or indefinite, induces significant changes in altruism or the concern for one-sided fairness. Table 4 reports estimation results using only repeated game data from the R1- and R2-tasks. It contains the estimates for the three parameter vector $\delta^\eta, \eta \in \{\beta, \gamma, \omega\}$ for the model that contains an intercept and a dummy for the finitely repeated game. All values are on the original scale. There are no significant effects of finite repetition on the estimated social preference paramet-
Table 4: Effects of the Type of Repetition (using R-task data)

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>15.02***</td>
<td>11.16***</td>
<td>0.31***</td>
</tr>
<tr>
<td></td>
<td>(1.80)</td>
<td>(1.39)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>Finitely Repeated</td>
<td>4.92</td>
<td>3.63</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(8.74)</td>
<td>(6.47)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.01***</td>
<td>1.75***</td>
<td>1.64***</td>
</tr>
<tr>
<td></td>
<td>(0.0057)</td>
<td>(0.016)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>N</td>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>-4962</td>
<td></td>
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</tbody>
</table>

Notes: *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$. Coefficients are transformed back to the original scale. A treatment effect is the (partial) effect of moving from the baseline treatment to the corresponding treatment on the median parameter value. Standard errors are calculated using the sandwich estimator and treating all of each subject’s choices as a single super observation. The entries for $\sigma$ are the standard deviations of the untransformed normal distributions of the random coefficients.

6.4 Predicting repeated game choices

One of the characteristics of choices from repeated public goods games is the substantial variation in individual level behavior within round and across rounds. Only examining aggregate choices will miss this variation. Models of individual behavior should be able to explain variation at the individual and aggregate levels.

We evaluate to what extent social preference parameters estimated with the P- and R-data are able to explain heterogeneous contribution dynamics at the individual level in the repeated games across costs to cooperate. Thus the estimated models can be evaluated on their effectiveness in fitting individual level choices and also predicting choices in the corresponding repeated game. We can also examine if decisions in a one-shot environment for a given MPCR predict decisions in a repeated environment for the other MPRC. We are not aware of other papers that have done this type of analysis in the context of public

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22Appendix 9 shows that belief formation is also identical across finitely and indefinitely repeated games in the regression framework used in Fischbacher and Gächter (2010).

23For example, a model of a simple monotonically decreasing curve matches average contributions of individuals well in finitely repeated public goods games with a small MPCR. While such a model could explain average choices, it would have a difficult time explaining contribution dynamics at the individual level.
goods games. For a similar analysis in this context of 2 × 2 games see Chmura, Goerg, and Selten (2012).

To conduct the analysis, we make use of the individual level “posteriors” of the structural parameters by using an approach similar to that of Revelt and Train (2000). To calculate posteriors, we use the estimates from the pooled data (one-shot and repeated) reported in Table 3. For a given MPCR, experience and type of game, the density \( f(\eta_i|X_i, \hat{\delta}, \hat{\Sigma}, g_\eta) \) represents the population level distribution of \( \eta_i \) for the treatment with characteristics given in \( X_i \). Using this distribution as a “prior,” one can infer the individual level ”posterior” distribution of \( \eta_i \) of an individual in a treatment with characteristics \( X_i \) by conditioning on his/her observed choices in that treatment.

In the following we evaluate the success of the (expected) posteriors of the structural parameters of in explaining the choices in repeated games.

6.4.1 Measure of predictive success

The predictive success of posteriors is measured by applying the quadratic scoring rule for repeated game choices of the 150 participants. The quadratic scoring rule was first introduced by Brier (1950), and axiomatically characterized in Selten (1998). Let, for a participant \( i \) in round \( t \) of the repeated game, a given decision making model predicts each contribution level \( j \) with a probability \( p_j(t) \). If a participant’s observed contribution level in round \( t \) is \( k \), then the quadratic score of the decision making model in round \( t \) is given as:

\[
\text{Quadratic score} = k - p_j(t)
\]

The logarithmic scoring rule could be used instead of the quadratic scoring rule to evaluate predictive success. That is, if posteriors predict an observed choice with a probability \( p \), then the corresponding logarithmic score is \( \log(p) \). We prefer to use the quadratic score because it is bounded. Furthermore, Selten (1998) notes that the logarithmic score has an undesirable property of being hypersensitive.
Thus, in computing the quadratic score, the observed choice is interpreted as a degenerate probability distribution with the observed contribution having a probability of 1 and all other contributions having a probability of 0. The quadratic score ranges between [-1, +1]. Higher scores indicate the relative success of a model in predicting the observed choices.

For each participant, we compute the quadratic score for each round in a repeated game given the posterior values of the structural parameters. We take the average of these quadratic scores, \( \bar{q} \), as a measure of the predictive success of the given posteriors in explaining contributions in the repeated game. If there are \( L \) rounds in a repeated game, then the Mean Quadratic Score (MQS) for the repeated game is:

\[
\bar{q} = \frac{\sum_{l=1}^{L} q(l)}{L}
\]

Before proceeding to evaluate the predictive success of the estimated posteriors, we introduce a basic benchmark. This benchmark is the score obtained by choosing randomly among all possible contribution levels. Since there are 21 possible contribution levels available in each round, such a purely random model would receive a quadratic score of 0.0476 in each round of a repeated game.

### 6.4.2 Mean quadratic scores with stated beliefs

Figure 2 presents the average individual level MQS for three models, ALPRbase, ALPRnoise and ALRR. The prefix AL refers to the Arifovic-Ledyard model that we are using to specify social preferences. The PR refers to the use of the expected values of the posterior distributions of structural parameters computed by conditioning on choices in the P-task to predict choices in the corresponding R-task. ALPRbase and ALPRnoise uses posteriors calculated from estimates reported in Table 3. ALPRnoise directly addresses mismatch in noise levels. ALRR examines prediction of choices in repeated games using posteriors computed by conditioning on choices in R-tasks. The bars in the graph are labeled R1 and R2, and they refer to decisions in the first repeated game and the second repeated game respectively (for an individual, each of the repeated games is conducted at a different level of MCPR). The red horizontal line indicates the random choice benchmark.

Looking first at ALPRbase, for both R1 and R2, average MQS across individuals lie significantly above the random choice benchmark (Paired Hotelling’s \( T^2 \) test (Johnson,

\[ q(t) = 2p_k(t) - \sum_{j=0}^{20} p_j^2(t) \]
Wichern, et al., 1992), \( p = 0.0083 \). This says that the choices in the strategy environment nontrivially inform behavior in a repeated environment. However, Table 3 shows that social preference parameters are stable across strategy and repeated games but individuals tend to be more than ten times noisier in repeated environments. This could be a reason for the relatively small MQS scores of ALPRbase since it uses the posterior noise parameter from the strategy games along with the posterior social preferences. In ALPRnoise, we address this mismatch in noise levels by using the posterior social preference parameters from the strategy game and the posterior noise parameter computed from choices in the corresponding repeated game.\(^\text{30}\) The average MQS goes up in both R1 and R2 in ALPRnoise compared to ALPRbase and is significantly larger (Paired Hotelling’s \( T^2 \) test, \( p = 0.0275 \)).

ALRR evaluates in-sample predictive success of the posteriors in each of the repeated games. Here the MQSs of choices in R1 are computed using the posteriors obtained from R1 and the MQSs of choices in R2 are computed using the posteriors obtained from R2. Intuitively, the MQSs should be larger in this case since we are using the posteriors computed using the choices in a repeated game to predict the same choices. The pair of MQS scores in ALRR are significantly larger than that of the random choice benchmark (paired Hotelling’s \( T^2 \) test, \( p = 0.0003 \)) and ALPRbase (paired Hotelling’s \( T^2 \) test, \( p = 0.0219 \)). However, they are not different from the MQS scores in ALPRnoise (paired Hotelling’s \( T^2 \) test, \( p = 0.292 \)). The result is interesting because it confirms that choices in the repeated game do not add additional information about social preference parameters than what is learned from choices in the corresponding strategy game. This

\(^\text{30}\)The MQSs of R1 choices are computed by using the posteriors of \( \beta, \gamma \) computed using responses in P1 and the posterior \( \omega \) is computed from choices in R1 itself. A similar procedure is used for choices in R2.
is consistent with the aggregate estimations reported in Table 3.

6.4.3 Stated beliefs vs estimated beliefs

Thus far, we have made the assumption that individuals best respond to their stated beliefs in repeated games. One justification for this approach is that our belief elicitation process was incentivized for participants to report accurate beliefs. However, it is possible that participants are best responding to beliefs formed in a potentially different way and these are substantially different from stated beliefs. We address this alternative in the following.

We consider one of the standard belief formation models proposed by Cheung and Friedman (1997). To distinguish from stated beliefs, we call beliefs formed in this manner as estimated beliefs. Cheung and Friedman (1997) propose a one-parameter belief formation model that includes Cournot and fictitious play beliefs as special cases. In their approach, a free-parameter $\kappa_i \in (0, +\infty)$ represents the individual level discounting of previous information.\(^{31}\) A participant $i$’s belief that others in his group contribute an amount $a_j$ in period $t + 1$ is computed as:

$$b_{i(t+1)}^j = \frac{\mathbb{I}_t(a^j) + \sum_{u=1}^{t-1} \kappa_i^u \mathbb{I}_{t-u}(a^j)}{1 + \sum_{u=1}^{t-1} \kappa_i^u}$$

where $b_{i(t+1)}^j$ is participant $i$’s belief about the likelihood that the average contribution of his group members is $a^j$ in period $t + 1$. $\mathbb{I}_t(a^j)$ is an indicator function equal to 1 if $a^j$ was the average contributions of others in period $t$ was $a^j$ and 0 otherwise. Likewise, $\mathbb{I}_{t-u}(a^j)$ is equal to 1 if the average contribution of others was equal to $a^j$ in round $t - u$ and 0 otherwise. We apply the belief learning rule beginning at $j = 1$ and to reduce the number of free parameters we neglect any beliefs held prior to the round 1. Setting $\kappa_i = 0$ yields the Cournot belief learning rule, and setting $\kappa_i = 0$ gives rise to the fictitious play type of beliefs. We have adaptive learning when $0 < \kappa_i < 1$; in this case all observations influence the expected state of beliefs but the more recent observations have a greater weight. Finally, when $\kappa_i > 1$, older observations have a greater weight and characterize the belief formation of an individual who relies on first impressions.

Figure 3 presents the average of individual level MQS’s in R1 and R2 for the two models ALPRbase and ALPRnoise that were considered earlier, but here estimated beliefs are used rather than stated beliefs. At the individual level, we estimated the $\kappa_i$ that maximized the log-likelihood of choices in a given repeated game and use it in the computation of MQS of the same. The pair of MQS scores computed using ALPRbase at the individual level are not different from the random choice benchmark (paired Hotelling’s $T^2$ test, $p = 0.129$), but they are significantly smaller than those computed using ALPRbase with stated beliefs (paired Hotelling’s $T^2$ test, $p = 0.0020$). For ALPRnoise with estimated beliefs, individual level MQS scores are larger than that of random choice benchmark.

\(^{31}\) In their original presentation, Cheung and Friedman (1997) assume $\kappa \in (-\infty, +\infty)$. However, values of $\kappa_i < 0$ are counterintuitive in that they imply influence of a given observation changes sign each period. We preclude this possibility in our estimations.
(paired Hotelling’s $T^2$ test, $p = 0.091$) but smaller than that of the MQS scores computed using ALPRnoise using stated beliefs (paired Hotelling’s $T^2$ test, $p = 0.0001$).

These results suggest that, to the extent that individuals are best responding to their beliefs, individuals are best responding to stated beliefs rather than estimated beliefs. This finding is in line with previous literature that compares stated beliefs and estimated beliefs in the context of two player and two strategy experimental games (Nyarko & Schotter, 2002).

6.4.4 Cross prediction

We turn to the generalizability of the structural parameter posteriors by examining cross prediction fit. Taking the three cases considered in Figure 2, we use posteriors computed from a strategy game with a given MPCR level to predict choices in a repeated game with a different MPCR level. What it means for ALPRbase is that we use posteriors computed from P1 to predict choices in R2. In ALPRnoise, we do the same as in ALPRbase but we use the posterior noise parameter from the repeated game. As for ALRR, we use the posteriors computed using R1 to predict choices in R2 and use the posteriors computed using R2 to predict choices in R1.

Figure 4 presents results for the three cases of ALPRbase, ALPRnoise, ALRR. For ALPRbase and ALRR, individual level MQS scores are worse than the random choice benchmark (paired Hotelling’s $T^2$ tests, $p < 0.0001, p = 0.0055$). For ALPRnoise, individual level MQS scores are no different from the random choice benchmark (paired Hotelling’s $T^2$ test, $p = 0.110$). What these results indicate is that the belief learning model with social preference parameters potentially overfits observed behavior in a given environment and is not easily generalizable to describe behavior across environments (involving changes in the cost to cooperate). The estimation results in Table 3 are suggestive of this. Experience and MPCR have a significant effect on the social preference parameters which indicates that the social preferences are shifting with changes in the environment. Thus, preference parameters that describe behavior in one environment
6.5 Comparing models of learning with repeated game data

Thus far, the results show that a belief learning model with stated beliefs better fits the choices in the repeated games. We would like to compare this with the two payoff-based learning models described in Section 4: the reinforcement learning model with payoff variability (REL), and the self-tuning experience-weighted attraction (STEWA) learning model. REL is a representative learning model for reinforcement learning and STEWA is a representative model that combines both belief learning and reinforcement learning. These models can help us examine whether belief learning, reinforcement learning or a mix of the two best explains choices.

6.5.1 Aggregate estimations

The three learning models are estimated using the aggregated repeated game data. The estimation methodology follows the description in Section 5. Table 5 reports parameter estimates of each of the models and the corresponding log-likelihood.

First we note that the estimates of the AL model are similar to those in Table 3 and thus offer no significant advantage of conducting a separate estimation with just the repeated game data. The estimated parameters of REL in the second panel of the table show the presence of significant heterogeneity in the structural parameters (the estimated standard deviations of the parameters are significantly different from zero). The median noise parameter, $\lambda$, indicates the presence of a moderate amount of noise in decision making. The estimates of $\lambda$ in the STEWA model indicate the presence of a large amount of noise in decision making. The median value of $\lambda$ is not significantly different from zero and indicates a median response not different from random choice when one assumes STEWA learning. However, there is a presence of a large amount of heterogeneity in the noise parameter as indicated by a large standard deviation.
Table 5: Estimation of Models with No Covariates Using Repeated Game Data

<table>
<thead>
<tr>
<th>AL (β)</th>
<th>REL (λ)</th>
<th>STEWA (λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median (β)</td>
<td>Median (λ)</td>
<td>Median (λ)</td>
</tr>
<tr>
<td>17.92***</td>
<td>1.08***</td>
<td>0.0012</td>
</tr>
<tr>
<td>(0.15)</td>
<td>(0.060)</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>σβ</td>
<td>σλ</td>
<td>σλ</td>
</tr>
<tr>
<td>2.01***</td>
<td>0.39***</td>
<td>3.67***</td>
</tr>
<tr>
<td>(0.0037)</td>
<td>(0.062)</td>
<td>(0.44)</td>
</tr>
<tr>
<td>Median (γ)</td>
<td>Median (N1)</td>
<td></td>
</tr>
<tr>
<td>13.28***</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>(0.083)</td>
<td>(0.48)</td>
<td></td>
</tr>
<tr>
<td>σγ</td>
<td>σN1</td>
<td></td>
</tr>
<tr>
<td>1.76***</td>
<td>4.05***</td>
<td></td>
</tr>
<tr>
<td>(0.0041)</td>
<td>(0.46)</td>
<td></td>
</tr>
<tr>
<td>Median (ω)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.036)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σω</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.67***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.19)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N 150 150 150
No. of Parameters 6 4 2
LL -4964 -4903 -6317
BIC 9958 9826 12644

Notes: ∗p < 0.1, ∗∗p < 0.05, ∗∗∗p < 0.01. Coefficients are transformed back to the original scale. A treatment effect is the (partial) effect of moving from the baseline treatment to the corresponding treatment on the median parameter value. Standard errors are calculated using the sandwich estimator and treating all of each subject's choices as a single super observation. The entries for σ are the standard deviations of the untransformed normal distributions of the random coefficients.

To assess the fit among the various learning models, we use several model selection criteria such as the parametric Youg's test (Vuong, 1989), Clarke's distribution-free test (Clarke, 2007), and the information criteria like AIC, BIC (Schwarz et al., 1978). The best log-likelihood is achieved by the REL model, and REL is preferred over STEWA based on all model selection criteria considered (Vuong Test: z = 8.40, p < 0.0001, Clarke Test: C = 25, p = Binomial(25, 150, 0.5) < 0.0001, BIC is smaller for REL). However, a comparison between REL and AL is not as straightforward. This is because AL uses the data on stated beliefs in addition to the data on contributions in the repeated games. Thus, AL uses more data than REL and this makes it difficult to compare them using model selection tests such as Youg's or Clarke's. Despite the advantages that the AL model has in fitting the data, the REL model has a larger log-likelihood.

We conclude that the REL and AL models outperform STEWA in explaining the repeated game data, and the REL model performs at least as well as AL in aggregate estimations.

6.5.2 Predictive success of learning models at the individual level

We now turn to a comparison of the different learning models in explaining individual level contribution dynamics. Using the calculated posterior parameters of each learning model, we examine its in-sample and out-of-sample predictive success. The posteriors
are computed as described in Section 6.4. In the analysis of in-sample predictive success, we compute MQS of choices in R1 using the posteriors computed using R1 choices and compute MQS of choices in R2 using the posteriors computed using R2 choices. Out-of-sample predictive success is evaluated through the computation of MQS of R1 choices using the posteriors computed from R2 choices and the computation of MQS of R2 choices using the posteriors computed with R1 choices.

Figure 5 presents the average of in-sample individual level MQS scores computed for each of the learning models. AL and REL models perform much better than a random choice benchmark and STEWA (Paired Hotelling’s $T^2$ tests, $p < 0.0001$). STEWA does not perform any better than the random choice benchmark (Paired Hotelling’s $T^2$ tests, $p = 0.210$). Individual level MQS scores are not different across AL and REL (Paired Hotelling’s $T^2$ tests, $p = 0.985$). These results agree with the general picture that one gets by looking at the log-likelihood of models in Table 5.

Out-of-sample prediction paints a very different picture. Here AL does not have a clear advantage over STEWA or the random choice benchmark (Paired Hotelling’s $T^2$ tests, $p = 0.146$ for $\{\text{AL, Random Choice}\}$ and $p = 0.48$ for $\{\text{AL, STEWA}\}$). This highlights the results observed in the previous subsection that AL overfits to the data and does not explain behavior across changes in MPCR and experience. This is not the case with REL. REL achieves significantly higher MQS scores compared to all other models in cross prediction and performs significantly better than a random choice benchmark (Paired Hotelling’s $T^2$ tests: $p < 0.0001$ for all pairwise tests $\{\text{REL, AL}\}$, $\{\text{REL, STEWA}\}$, $\{\text{REL, Random Choice}\}$).

These results suggest that a reinforcement learning model with a simple payoff-based learning explains aggregate repeated game data and individual level contribution dynamics significantly better than that of the social preference based belief learning model with stated beliefs and the pay-off based STEWA that mixes belief learning and reinforcement learning.
6.6 Explaining repeated game decisions: social preferences, learning or both?

Our analysis thus far has found that the payoff-based REL model provides a better fit to the data at the aggregate and individual level. Cross prediction at the individual level shows that the REL model does not overfit the data as a model of social preference based belief learning. Furthermore, REL does not require elicitation of beliefs or any assumption of belief formation. Thus, behavior in the repeated games is more in line with a parsimonious reinforcement learning model in which individuals care only about their own payoffs in the game.

An equally important observation from the repeated game data is that there are substantial differences in contributions across behavioral types. For example, using the type classification with posteriors of social preferences based on choices in the P1-task (types are determined from posterior social preference parameters using the computations in Appendix 3), the average contribution of free riders, conditional cooperators, full cooperators, and noisy contributors are 7.83, 12.74, 14.56, and 11.32 respectively in the corresponding R1-task.\footnote{Using the LCP classification, the average contribution of free riders, conditional cooperators, full cooperators, and noisy contributors are 7.53, 12.45,16.14, and 10.93 respectively in R1. The contributions are significantly different across types (Kruskall-Wallis Test: \( p < 0.0001 \)), and there is a significant increasing trend in contributions across the groups (Jonckheere-Terpstra Test, \( p < 0.0001 \))} Similarly, using the type information from the P2-task, the average contributions are 6.30, 12.04, 13.77, and 11.68 respectively in R2.\footnote{Using the LCP classification, the average contributions are 6.30, 12.28, 15.28, and 8.11 respectively. The contributions are significantly different across types (Kruskall-Wallis Test: \( p < 0.0001 \)), and there is a significant increasing trend in contributions (Jonckheere-Terpstra Test, \( p < 0.0001 \))} This indicates that social preferences elicited in the P-tasks have a nontrivial relevance in explaining average contributions in the repeated games. However, a payoff-based REL learning model alone cannot explain the significant differences in contribution behavior in the repeated game across behavioral types.
In the following, we shed light on how these two findings could hold at the same time. To begin, we compare the first round contributions in the repeated games and the unconditional contributions in the strategy games and find they are not significantly different from each other (Paired Hotelling’s $T^2$ test, $p = 0.118$), but the contribution choices of each type are significantly different within each game (Kruskal Wallis Tests: $p < 0.0001$ in all cases). Thus the first round contributions in the repeated game are similar to the unconditional contributions in the corresponding strategy game and are informed by individual level social preferences derived from the strategy game. In the first round of the repeated game, free riders tend to start with contribution that are significantly smaller on average (though not exactly zero) and full contributors start with significantly higher contributions on average (though not exactly full endowment). The remaining participants start around half of the endowment.

The first round contributions in the repeated game reflect the social preferences of the individuals just like unconditional contributions in the strategy games do. Although reinforcement learning does not distinguish across “types” to determine the levels of contributions in each round, the initial attractions of the strategies are derived from the empirical distribution of first round contributions (see Appendix 4). First round contributions in repeated games are generally multi peaked with a significant mass around zero, half the endowment, and the full endowment. Initial attractions reflect this and thus carry the information about the social preferences of individuals. This leads to the question of whether social preferences also inform contribution decisions in the later rounds along with learning. If so the REL model that uses utilities based on social preferences to update attractions would fit the data significantly better. Otherwise, we can conclude that contributions in the later rounds are solely determined by payoff-based learning. To disentangle the role of social preferences and payoff-based learning, we use agent-based simulations that allow for counter factual assumptions about preferences and learning.

In our simulations, we generate individual level contributions according to a learning model and social preferences given the actual contributions of the other group members in each round. Since the decision making process in the learning model is probabilistic, we conduct 500 simulations per individual and compute the average path of contributions. To measure how close the simulated path of contributions is to the observed path at the individual level, we calculate the Root Mean Squared Error (RMSE) between the path of actual contributions and the simulated average contribution path. For each individual, we have two measures of error, one for each of the repeated games.

To be able to disentangle the roles of social preferences and learning, we consider five models that involve different assumptions about learning and social preferences. In each of these models, we use posterior social preference parameters computed using the strategy games for the corresponding repeated games wherever necessary. The models

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34 There is also a significant increasing trend in first round contributions in the repeated games across free riders, conditional cooperators and full cooperators (R1: Jonckheere-Terpstra Test, $p < 0.0001$; R2: Jonckheere-Terpstra Test, $p < 0.0001$). The trend is also significant for unconditional contributions in the strategy games across these three types (P1: Jonckheere-Terpstra Test, $p < 0.0001$; P2: Jonckheere-Terpstra Test, $p < 0.0001$).
considered are listed below.

- REL-RPB: reinforcement learning based on payoffs in the game with random first round contributions. This model does not use any information about the social preferences of the participants. In the learning phase, individuals use payoffs in each round to update attractions of strategies in the REL learning model.

- REL-SPB: reinforcement learning based on payoffs in the game with first round contributions determined from the social preference parameters of the participants. The first round contribution of an individual is determined using the stochastic best response (using logit probability) to a random belief about others’ contributions that is drawn from \( U[0, w] \). In the learning phase, individuals use payoffs in each round to update attractions of strategies in the reinforcement learning model.

- REL-RUB: In this model individuals start with random contributions. In the learning phase, individuals use utilities in each round computed using posterior social preferences to update attractions of strategies in the reinforcement learning model.

- REL-SUB: This is identical to REL-SPB except that in the repeated game attractions are updated using utilities computed using social preferences rather than simple payoffs.

- IEL: Individual Evolutionary learning (IEL) of Arifovic and Ledyard (2012) starts with random contributions and uses utilities based on social preferences from the contributions in the three steps of learning: experimentation, replication, and selection. We use IEL to test for the possibility that learning mechanism used by the participants may be better explained by a learning model of an evolutionary nature rather than by a learning model with cognitive foundations like that of REL and STEWA.\(^{35}\)

To reduce the dimension of the estimation, we fixed the initial attractions at zero in the REL model. We also assume that the initial strength of attractions is \( N(1) = 0 \).\(^{36}\) Since we allowed for heterogeneity in the learning parameters of the REL model in our aggregate estimations, we estimated the noise parameter using a grid search over the interval of [0 10] for each individual separately that resulted in the smallest RMSE for each game. For the IEL model, no parameters were estimated. We used the learning parameters \((J, \rho, \sigma) = (100, 0.033, w/10)\), \( w \) is the endowment, as used in Arifovic and Ledyard (2012). The simulations by Arifovic and Ledyard (2012) show that the results are quite robust to changes in these parameters. For social preference parameters in IEL, we used the posterior social preference parameters computed using the corresponding strategy game.

\(^{35}\)We did not consider IEL model in Section 6.5 since IEL generates a Markov process on the subsets of strategies rather than on strategies themselves making it difficult to estimate with standard statistical techniques like maximum likelihood.

\(^{36}\)This reduces the number of parameters estimated at the individual level. Furthermore, the median estimated parameter of \( N(1) \) is not significantly different from zero in our estimations reported in Table 5. The initial accumulated payoffs are fixed at zero, and the initial payoff variability is fixed at one.
The results of these simulations are shown in Figure 7. The smaller the RMSE is the better the fit. The largest RMSEs are found for the REL-RPB (R1: 5.66, R2: 6.05) and IEL (R1: 5.57, R2: 5.98) models, and these are not different from each other (Paired Hotelling’s $T^2$ test, $p = 0.459$). The importance of first round contributions can be seen by comparing the average RMSEs in REL-RPB and REL-SPB (R1: 4.77, R2: 4.60). Across these two models, the learning process is the same, but first-round contributions are determined differently. When first round contributions are determined by social preferences from the corresponding strategy game and learning is payoff-based, as in the REL-SPB model, the RMSE is significantly smaller than REL-RPB, IEL and REL-RUB (Paired Hotelling’s $T^2$ tests, $p < 0.0001$). The significant difference between REL-SPB and REL-RUB highlights the fact that initial contributions determine the path of the play, not the utilities derived from social preferences in each round that are used to update the attractions of the strategies. This result stands in contrast to the assumption in the literature that individuals start with random contributions and move towards their equilibrium contributions based on learning with utilities derived from social preferences (Cooper & Stockman, 2002; Janssen & Ahn, 2006; Wendel & Oppenheimer, 2010; Arifovic & Ledyard, 2012). Finally, the RMSEs are not different across REL-SPB and REL-SUB (R1: 4.80, R2: 4.62) (Paired Hotelling’s $T^2$ test, $p = 0.306$). This result shows that once initial round contributions are determined by social preferences, there is no additional information gained from social preferences to explain the path of play. In other words, social preferences matter insofar as they determine first-round contributions and then the choices of individuals are consistent with a simple model of payoff-based learning.

Free riders contribute on average less than other behavioral types in the repeated games, but their contributions are significantly larger than a zero. An interesting candidate explanation for this observation is that free riders are behaving strategically (Muller et al., 2008; Ambrus & Pathak, 2011). In an experiment where the type information of the group one belongs to is common knowledge, Ambrus and Pathak (2011) find that free
riders reduce their contributions significantly earlier than that of others, thus providing evidence for strategic motives. We test this proposition with our data by computing the number of the round in which the drop in contributions is the largest and testing if the round number is significantly smaller for free riders compared to others. We do not find a significant difference in either R1 or R2 games (R1: Mann-Whitney test, $p = 0.299$; R2: Mann-Whitney test, $p = 0.951$) or when we consider games with finite and indefinite repetition separately (finitely repeated: Mann-Whitney test, $p = 0.487$; indefinitely repeated: Mann-Whitney test, $p = 0.796$). Thus we do not find evidence that free riders contribute larger than zero in the first round because they are strategic. This further supports our finding that behavior in the repeated games is more in line with learning based on payoffs rather than strategic motives.

It is also equally important to note that while full contributors contribute significantly higher amounts in the first round of a repeated game compared to other types, their contributions are significantly smaller on average than the full endowment. The underlying reason for these patterns of first round contributions of free riders and full contributors could be potentially their resistance to use extreme strategies (Kümmerli et al., 2010).

7 Conclusion

This paper investigates contribution decisions to a linear public good as the cost to cooperate and the duration of interaction change. We ask whether changes in decisions across these environments reflect stable underlying social preferences or not. Using data collected from laboratory experiments, we structurally estimate a model of social preferences that accounts for costs to cooperate and controls for decision error to estimate preferences for cooperation. The results provide mixed evidence on preference stability. In a one-shot environment, the distribution of estimated social preference parameters is significantly different as the cost to cooperate changes. That is, underlying preferences are not stable as costs change. However, to the extent that individuals are myopic belief learners who best respond to their beliefs about others’ contributions in each round of repeated games, the distributions of social preferences are identical across one-shot and repeated games. Furthermore, the type of repetition, finite or indefinite, does not have a significant effect on belief formation or the distribution of social preferences. To the extent that individuals best respond to their beliefs we find that stated beliefs do a better job at explaining repeated game choices compared to the estimated beliefs.

Our individual level analysis indicates that belief learning with social preferences overfits the data and cannot generalize to explain contribution dynamics across changes in cooperation costs. Comparing a number of representative models of learning shows that a simple payoff-based reinforcement learning has the best fit when explaining aggregate repeated game data and contribution dynamics at the individual level.

\[\text{In fact, we do not find differences across any types in either R1 or R2 games (R1: Kruskal-Wallis test, } p = 0.390; \text{ R2: Kruskal-Wallis test, } p = 0.962) \text{ or when we consider games with finite and indefinite repetition separately (finitely repeated: Kruskal-Wallis test, } p = 0.329; \text{ indefinitely repeated: Kruskal-Wallis test, } p = 0.792).\]
A number of previous studies have recognized that both learning and social preferences are important in explaining the contribution patterns in repeated public goods games (Cooper & Stockman, 2002; Janssen & Ahn, 2006; Wendel & Oppenheimer, 2010; Arifovic & Ledyard, 2012). These studies assume individuals start with random contributions and move towards equilibrium contributions determined by their social preferences. This is inconsistent with the choices made in the repeated games by the participants in our study. First round contributions are not random and do correlate with individual social preferences. We find that a hybrid model in which first round contributions are determined by social preferences and after that participants use a payoff-based reinforcement learning to make contribution decisions best reconciles choices across the one-shot and repeated environments.

Voluntary cooperation is inherently fragile. Mechanisms like costly punishment have been shown to sustain cooperation (Fehr & Gächter, 2000; Selton, Shupp, & Walker, 2007), however, such mechanisms in addition to being costly to implement can also lead to undesirable outcomes of antisocial behavior (Herrmann, Thöni, & Gächter, 2008). Our findings show that individuals are identical in the manner in which they learn, no matter what their underlying social preferences might be, but they differ in terms of the contributions that they start with. This suggests that higher first round contributions might lead to sustained levels of cooperation over time. However, more research is needed to understand how such behavior might be encouraged and the effects on contribution dynamics.
References


Palfrey, T., & Prisbrey, J. (1997). Anomalous behavior in public goods experiments:


Appendix

1 Instructions
INSTRUCTIONS

Welcome and thank you for participating in today’s economic experiment. Please put away all your belongings and turn off your cell phones. You are not allowed to talk to any other participant during the experiment. If you have any questions, please raise your hand. We will come to you and answer your questions in private. The experiment will be run entirely on the computer and all interactions between yourself and others will take place via the computer terminal.

You have earned $5 just for showing up on time. This is yours to keep. In addition, depending upon the decisions you make, the decisions others make and random choice, you can earn more money. These instructions describe in detail the experiment and tasks you are asked to complete.

During the experiment, your earnings will be described in terms of tokens. At the end of the experiment, the total number of tokens you have earned will be converted to money at the following rate:

\[20 \text{ tokens} = \$1 \quad (1 \text{ token} = 5 \text{ cents})\]

The experiment consists of four tasks. You will receive instructions for each task prior to making decisions for that task. Your total earnings from the experiment will be the sum of your earnings in each task. At the end of the session, the total number of tokens earned across all four tasks will be converted to money and paid to you privately in cash, along with the $5 show-up fee.
Your Neighborhood

You will be placed in a network with 14 other participants as shown in Figure 1. The placement of participants in the network is random, and participants do not know who is connected to whom.

Each participant is placed at one position on the network and is connected to exactly two other participants. This placement and connection are fixed throughout each of the four tasks. You and the two other participants who are connected to you in the network define your “neighborhood”. In Figure 1 – for example, if you are placed in the position of the circle that is highlighted in pink then your “neighbors” are highlighted in yellow. In the network in Figure 1, there are three connected participants in each neighborhood and five neighborhoods.

Figure 1: Network
The Decision Situation

Each participant is provided with 20 tokens and must decide how to allocate the tokens between a private account and a group project. You can choose to put none, all or some of your tokens into the group project. The tokens you choose not to contribute to the group project will be put in your private account.

For each token put in your private account you earn exactly one token. You are the only one who earns tokens from your private account.

What you earn from the group project depends on the total number of tokens that you and your neighbors contribute to the group project. The more each member of the neighborhood contributes to the group project, the more each member earns. Remember that your neighborhood includes you and two other participants.

Your earnings from the group project are best explained by a number of examples.

**Example 1:** Suppose that you decided to contribute no tokens to the group project but the 2 other members of your neighborhood contribute a total of 36 tokens. Then your earnings from the group project would be 36 tokens x 0.4 = 14.4 tokens. Everyone else in your neighborhood would also earn 14.4 tokens.

**Example 2:** Suppose that you contribute 15 tokens to the group project and the 2 other members of your neighborhood contribute a total of 36 tokens. This makes a total of 51 tokens in your neighborhood. Your earnings from the group project would be 51 tokens x 0.4 = 20.4 tokens. Everyone else in your neighborhood would also earn 20.4 tokens.

**Example 3:** Suppose that you contribute 20 tokens in the group project but the other 2 members in your neighborhood contribute nothing. Then you would earn 8 tokens from the group project (20 tokens x 0.4 = 8 tokens). Your neighbors would also earn 8 tokens from the group project.

As you can see, every token contributed to the group project earns 0.4 tokens for every member of the neighborhood, not just the participant who puts it there. *It does not matter who contributes tokens to the group project. Everyone will get a return from every token contributed there—whether they contributed tokens in the group project or not.*

Your total earnings from the private account and group project will be:

**Your total earnings = 20 - your tokens contributed to the group project + 0.4 × sum of tokens contributed to the group project by all members of your neighborhood**

You will now complete some questions to make sure everyone understands how earnings are calculated.
Instructions for Task 1 – A-Task

In the A-task, you will make a choice for the decision situation described earlier. You will have 20 tokens and must decide how many to put into your private account and the group project. You will be randomly assigned to a 3-participant neighborhood.

Each participant has two decisions in this task: make an unconditional contribution and complete a contribution table. Details about these two decisions are as follows.

**Unconditional Contribution:** In this decision, you must decide how many of the 20 tokens you would like to put in the group project. You will make your decision on a screen such as the following.
**Contribution Table:** In this decision, you must decide how many tokens you would like to contribute to the group project for every possible average contribution of your neighbors (e.g. 0, 1, 2, …, 20). That is, if your neighbors contributed 0 tokens on average, how much would you contribute? If they contributed 1 token on average, how much would you contribute? If they contributed 2 tokens on average, how much would you contribute? And so on, up to 20 tokens contributed on average.

This means that in total you have to give 21 responses. You will make your decisions on a screen such as the following.
Once each participant has made the unconditional decision and completed the contribution table, the computer will randomly determine if the unconditional contribution or the contribution table will be used to determine your earnings. In each neighborhood, one of the three participants is randomly chosen to have the contribution table count to calculate earnings. For the other two participants in the neighborhood, the unconditional contribution counts to calculate earnings. How is this done? If the participant is chosen to have his contribution table count for earnings, first the unconditional contributions to the group project of his neighbors are averaged and rounded to the nearest integer (e.g. 0, 1, 2, ..., 20). Then, the contribution table of the participant is used to determine how many tokens the participant contributes to the group project. The number of tokens he contributes is the amount he specified for that particular average contribution of his neighbors.

For example, if the average contribution of his neighbors is 16 tokens (32 tokens/2), and he specified 10 tokens if the average contribution of his neighbors is 16, the total contributed to the group project by everyone in the neighborhood would be 42 tokens.

You will not know in advance which decision, the unconditional contribution or the contribution table, will count to determine your earnings, so you should make each decision carefully as though it will count.

The following examples should help make this procedure clear.

**Example 1:** Suppose the contribution table was randomly chosen to count for you. This means that the decisions you made in the contribution table determine your earnings. For the other two neighbors their unconditional contributions determine their earnings. Suppose that the total contributions of the other two neighbors are 26 tokens, and the average contribution 13 tokens (26 tokens/2). In your contribution table, suppose you chose to contribute 4 tokens if the average contribution of your neighbors is 13, then your earnings for Task 1 would be: $20 - 4 + 0.4 \times (4 + 26) = 28$.

**Example 2:** Suppose the unconditional contribution was randomly chosen to count for your earnings. Also, suppose that the unconditional contribution of the neighbor who was not selected for the contribution table to count is 12. If your unconditional contribution is 20, then the average unconditional contribution in your neighborhood is 16 tokens ((20 + 12)/2). If the neighbor selected to have his contribution table count chose 18 tokens if the average contribution of his neighbors is 16, then your earnings are: $20 - 20 + 0.4 \times (20 + 12 + 18) = 20$.

Are there any questions before we begin?
Instructions for Task 2 – B-Task

In the B-Task, you will be randomly assigned to a 3-participant neighborhood as described earlier. Your neighbors in Task 2 may be different from your neighbors in Task 1, however, you will remain with the same neighbors for all decisions you make in Task 2.

The B-Task lasts for several rounds. The number of rounds is randomly determined. In each round, you face the basic decision situation described at the beginning of the experiment. After each round, there is an 85% probability that there will be one more round. So, for instance, if you are in round 2, the probability there will be a third round is 85% and if you are in round 9, the probability there will be another round is also 85%. How this works is as follows. After each round, the computer will randomly draw a number between 1 and 100 (e.g. 1, 2, 3, …, 100), where each number is equally likely to be chosen. If the chosen number is 85 or lower, there will be another round. If the chosen number is 86 or above, there will be no additional rounds, and the task will end. You will know there is another round if you see the decision screen again and are asked to make a decision. If the task ends, you will get a message saying the task is done. You will not know ahead of time for how many rounds you will make decisions.

In each round, you will be given 20 tokens and must decide how many tokens you would like to contribute to the group project and how many you would like to put in your private account. You will receive earnings only from the group project that involves participants in your neighborhood. Your earnings from your contribution decision in a given round are determined as:

Your total earnings = 20 - your tokens contributed to the group project + 0.4 × sum of tokens contributed to the group project by all members of your neighborhood

You will participate in the decision situation repeatedly with the same neighbors, until it is randomly determined that there are no more rounds.
You will make decisions on a screen such as the following:

Here is an example to explain how earnings are calculated in each round:

**Example 1:** Suppose you chose to contribute 4 tokens and your neighbors chose to contribute 28 tokens in total. Your earnings in that round would be: $20 - 4 + 0.4 \times (4 + 28) = 28.8$ tokens.

In each round, after you decide how much to contribute to the group project, you will be asked to guess the average contribution to the project (rounded to the nearest integer) of your two neighbors. You will receive tokens for the accuracy of your estimate. If your guess is exactly equal to the average contribution of your neighbors you will receive 3 tokens in addition to your earnings for that round. If your guess was off by 1 token, you will get 2 additional tokens. If your guess was off by 2 tokens, you will get 1 additional token. And, if your guess was off by 3 or more tokens, you will get 0 additional tokens.

When everybody in your neighborhood has completed the two decisions for the round,
you will be shown each of their contributions, the total contributions to the group project, the average contribution and your earnings for the round. You will only be informed of the contributions of those in your neighborhood. You will not be informed of contributions of participants in other neighborhoods.

Once all participants in the experiment have completed the two decisions and are told their earnings and the contributions of their neighbors for the round, the computer will randomly draw a number between 1 and 100 to see if everyone plays another round. If there is not another round, the task is done.

Are there any questions before we begin?
Instructions for Task 3 – A-Task

In Task 3, you will make two decisions again as you did in the Task 1 – A-Task. The difference between this task and Task 1 is that for each token contributed to the group project you, and the other two neighbors, will get 0.8 tokens back.

Things to remember:

1. You will be randomly placed on the network at the beginning of the task and assigned to a 3-participant neighborhood. Your neighbors in this Task 3 may be different than in the previous two tasks.
2. You will make two decisions.
3. The first decision, the unconditional contribution, is how many of your 20 tokens you want to contribute to the group project.
4. The second decision, completing the contribution table, is how many of your 20 tokens you want to contribute to the group project for every possible average contribution of your neighbors (e.g. 0, 1, 2, ..., 20).
5. Each token contributed to the group project will earn 0.8 tokens for each participant in the neighborhood.
6. One of the two decisions, the unconditional contribution or the contribution table, will be randomly chosen to determine earnings. You will not know ahead of time which decision will count.

Are there any questions before we begin?
Instructions for Task 4 – B-Task

In Task 4, you will make decisions again as you did in the Task 2 – B-Task. The difference between this task and Task 2 is that for each token contributed to the group project you, and the other two neighbors, will get 0.8 tokens back.

Things to remember:

1. You will be randomly placed on the network at the beginning of the task and assigned to a 3-participant neighborhood. Your neighbors in Task 4 may be different than in the previous tasks, however, you will remain with the same neighbors for all rounds in this task.
2. You will face the same decision situation for several rounds. You must decide how many of your 20 tokens you want to contribute to the group project.
3. The number of rounds is randomly determined. After each round, there is an 85% probability that there will be one more round.
4. If there is another round, you will see a decision screen to make another choice. If there is not another round, you will see a message saying the task is over.

Are there any questions before we begin?
2 Group Level Contributions and Beliefs

In experiments with Strangers’ matching, which randomly shuffle participants across groups in each round, the unit of interaction is all participants in a session in a repeated game. In contrast, the unit of interaction in our repeated games is a 3-person group since our design uses fixed groups in repeated games (the Partners’ matching). Therefore, contribution and beliefs patterns at a group level rather than the session level are more informative in understanding the contribution dynamics. Figures 8, 9, 10 and 11 provide plots of average contributions and beliefs across groups in the experiments. We have in total 50 distinct groups in R1 and R2 each. 25 of the groups involve high MCPR and 25 of the groups involve low MPCR in each case. We have split the groups according to the level of MPCR in the figures. Each group level plot contains information about group number, whether it is from R1 or R2, the type of repetition (I - Indefinitely repeated, F- finitely repeated), MPCR (0.4 or 0.8) and the composition of the group in the format of (no. of FR - no. of CC - no. of FC - no. of NC) in its title. The type information is computed using the classification algorithm that uses Linear Contribution Profile (Kurzban & Houser, 2005). For example, the title “G-4 R1-I (M = 0.4) (2-1-0-1)” refers to the contribution and belief plots of Group number 4 in R1 game which is indefinitely repeated and conducted at the MPCR level of 0.4. And the group composition is given by two free riders, one conditional cooperator, zero full cooperators, and zero noisy contributors. The x-axis label of each plot contains the information about the individual level first round contributions and corresponding types. The types are coded as FR-1, CC-2, FC-3, NC-4. For example, the x-axis label of the group number 4 in R1, is given as “round 1(0-20-8) types(1-1-2)”. This means that the first member of the group is a free rider, the second member is also a free rider, and the third member is a conditional cooperator. Their first round contributions are 0, 20, and 8 respectively.
Figure 8: Group Level Contributions and Beliefs in R1 - Low MPCR (0.4)
Figure 9: Group Level Contributions and Beliefs in R1 - High MPCR (0.8)
**Contributions**

**Beliefs**

Figure 10: Group Level Contributions R2 and Beliefs in - Low MPCR (0.4)
Figure 11: Group Level Contributions R2 and Beliefs in - High MPCR (0.8)
3 Arifovic-Ledyard Model of Social Preferences and Type Switches

Monetary payoff from contributing $c$

$$\pi^i = e^i - c^i + \alpha \sum_{j=1}^{N_g} c^j$$

Arifovic-Ledyard (2012) specification: Utility derived by individual $i$ from contributing amount $c$ when others contribute $o$ on average:

$$u^i(c, o) = \pi^i(c, o) + \beta^i \bar{\pi}(c, o) - \gamma^i \max\{0, \bar{\pi}(c, o) - \pi^i(c, o)\}$$

Where $\beta^i \geq 0; \gamma^i \geq 0; \bar{\pi} = \frac{\sum \pi^i}{N}$

From Equation 2 in Section 4, three equilibria are possible for a given combination of $\beta \geq 0$ and $\gamma \geq 0$ of an individual: Free Riding (FR), Conditional Cooperation (CC), and Full Cooperation (FC). We derive conditions for these three possibilities below for each MPCR level in our experiments:

- When MPCR = 0.4
  FR: $0 \geq 0.07\beta - 0.6$
  CC: $0.67\gamma \geq 0.07\beta - 0.6 \geq 0$
  FC: $0.67\gamma \leq 0.07\beta - 0.6$

- When MPCR = 0.8
  FR: $0 \geq 0.47\beta - 0.2$
  CC: $0.67\gamma \geq 0.47\beta - 0.2 \geq 0$
  FC: $0.67\gamma \leq 0.47\beta - 0.2$

According to the theory some of the transitions among types are not impossible (for example, a person cannot switch from being a Free Rider to a Conditional Cooperator when MPCR changes from 0.8 to 0.4).

In the following, we present parameter restrictions for each possible type transition between two levels of MPCR. The transitions that are not possible are identified in the list. This is followed by parameter restrictions for the cases when types remain stable across two MPCR levels.

- Type Switches
  - switching from FR - CC when MPCR changes from 0.4 and 0.8:
    $\beta \in [0, 8.57]$ and $\beta \in [0.42, 1.40\gamma + 0.42]$
4 Calculating the Initial Attractions

While learning models specify how the attractions are updated in each round and how the contribution decisions are made based on the attractions, they are not very clear on how the initial attractions are specified. Erev et al. (1999) used expected payoff from a random choice as initial attraction of each strategy for the REL model. However, we do not use this approach since it is inconsistent with the empirical data. The first round contributions in public goods games are not uniformly distributed but are multimodal with peaks at 0, half the endowment and full endowment. Thus assuming equal attractions for all levels of contributions would lead to a poorer performance of the learning models. Since the pure learning models have no particular way of explaining the first round behavior, in their paper Ho et al. (2007), authors recommend at least four different ways to initialize attractions in first round. These approaches include sophisticated theoretical methods like the cognitive hierarchy model (C. F. Camerer, Ho, & Chong, 2004) and a more empirical method of determining attractions from the first round choices. We choose the latter in this paper and use it for both REL and STEWA learning models. Specifically, attractions in first round $A_j(1), \forall j,$ are determined by using the actual observed frequency of contributions by all participants in the first period. The same initial attractions are used for all participants. Let the empirically observed frequency of contribution $j$ in the first period is $f^j$. Then initial attractions are recovered from the equations:

$$\frac{e^{\lambda A_j(1)}}{\sum_k e^{\lambda A_k(1)}} = f^j, j = 1, ..., m$$
Table 6: Separate Estimation of $\beta$, $\gamma$ and $\omega$

<table>
<thead>
<tr>
<th></th>
<th>Strategy Game Data</th>
<th>Repeated Game Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median ($\beta$)</td>
<td>10.23***</td>
<td>17.92***</td>
</tr>
<tr>
<td></td>
<td>(0.81)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>$\sigma_\beta$</td>
<td>3.09***</td>
<td>2.01***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.0037)</td>
</tr>
<tr>
<td>Median ($\gamma$)</td>
<td>11.45***</td>
<td>13.28***</td>
</tr>
<tr>
<td></td>
<td>(0.91)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>1.79***</td>
<td>1.76***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.0041)</td>
</tr>
<tr>
<td>Median ($\omega$)</td>
<td>0.12***</td>
<td>0.36***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.0036)</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>3.34***</td>
<td>1.67***</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.19)</td>
</tr>
</tbody>
</table>

N     | 150   | 150   |
LL    | -13625 | -4964 |

Notes: ∗$p < 0.1$, ∗∗$p < 0.05$, ∗∗∗$p < 0.01$. Coefficients are transformed back to the original scale. A treatment effect is the (partial) effect of moving from the baseline treatment to the corresponding treatment on the median parameter value. Standard errors are calculated using the sandwich estimator and treating all of each subject’s choices as a single super observation. The entries for $\sigma$ are the standard deviations of the untransformed normal distributions of the random coefficients.

Solving these equations one could recover attractions in first round for a given $\lambda$, the noise parameter, that maximizes the likelihood of first round contributions. The above equations can be rewritten as:

$$A_j(1) - A_k(1) = \frac{1}{\lambda} \ln(f_j) - \frac{1}{\lambda} \ln(f_k), j,k = 1, ..., m$$  \hspace{1cm} (6)

For identification, we fix the attraction of the strategy with the lowest frequency $A_j(1)$ to a constant value. The lowest frequency is zero in our data and we add a constant $W/m = \frac{1}{2\lambda}$, where $W$ is number of equilibria and $m$ are number of strategies, to all frequencies and renormalize them to avoid $\ln(0)$. So, we use $\tilde{f}_j = \frac{f_j + W}{\sum_k f_k + W}$ is place of $f_j$ to solve Equations 6 for $A_j(1)$’s. To be fair to both models, we use initial round attractions computed this way to compute likelihood of choices in the first round for both REL and STEWA.

5 Separate Estimation with Strategy and Repeated Games

Table 6 presents the estimates of the social preference parameters and the random choice probability parameter for strategy and repeated games separately.
Table 7: Identification Results for Log-Normal Specification of Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True</th>
<th>Recovered†</th>
<th>Bias</th>
<th>T-stat</th>
<th>Cohen’s d</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_\beta$</td>
<td>2.22 (9.21)</td>
<td>2.23 (9.80)</td>
<td>0.0073</td>
<td>0.15</td>
<td>0.022</td>
<td>0.11</td>
</tr>
<tr>
<td>$\sigma_\beta$</td>
<td>2.68</td>
<td>2.96</td>
<td>0.28</td>
<td>3.31***</td>
<td>0.47</td>
<td>0.43</td>
</tr>
<tr>
<td>$\mu_\gamma$</td>
<td>2.17 (8.81)</td>
<td>2.19 (9.30)</td>
<td>0.018</td>
<td>0.48</td>
<td>0.064</td>
<td>0.072</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>1.59</td>
<td>2.07</td>
<td>0.48</td>
<td>7.04***</td>
<td>0.99</td>
<td>0.46</td>
</tr>
<tr>
<td>$\mu_\omega$</td>
<td>-0.73 (0.33)</td>
<td>-0.92 (0.29)</td>
<td>-0.19</td>
<td>-6.94***</td>
<td>-0.98</td>
<td>0.072</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>1.61</td>
<td>1.64</td>
<td>0.024</td>
<td>1.15</td>
<td>0.016</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Notes:
1. *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$
2. In brackets transformed median value

6 Econometric Model - Empirical Identification

To investigate the empirical identification of the econometric model we setup in Section 5, we set up a Monte Carlo experiment. Following the procedure outlined in Cherchi and de Dios Ortúzar (2008), we simulated a collection of data sets following the decision making process outlined in Section 5. In this exercise, we generated artificial datasets involving 150 individuals making decisions across P1 and P2 tasks as described in Section 2. We have used the parameter estimates obtained using the minimal model estimation without any co-variates using only strategy game data as reported in Table 6 to generate artificial data sets. Monte Carlo procedure is as follows:

• For each data set $i$ in [1..50]

  1. for each individual $j$ in [1...150]

     - Draw $\beta_j, \gamma_j$ and $\omega_j$ from distributions specified by original parameters
     - Generate series of choice situation corresponding P1 and P2 experiments with MPCR levels 0.4, 0.8 respectively
     - In each choice situation
       * compute utilities of each contribution level using $\beta_j, \gamma_j$
       * Add EV1 error to the utility of each contribution and call it the observed utility
       * Draw a random number $r$ from Uniform[0,1]. if $r < (1 - \omega_j)$ choose the contribution level with highest observed utility . Otherwise, choose each contribution level with equal probability.

  2. Run estimation as outlined in Section 5 and collect the estimated parameters.

Table 7 summarizes results from the Monte Carlo experiment outlined above. It can be seen that the original parameters used in generating data are recovered very closely. However in three instances, $\sigma_\beta, \sigma_\gamma$ and $\mu_\omega$, the recovered parameters seem to be statistically significantly different from true parameter values. While the t-tests show
Table 8: Estimation of $\beta, \gamma$ and $\omega$ Using Strategy Experiments: Logistic-Normal Specification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Median</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>13.34***</td>
<td>5.28***</td>
<td>43.83***</td>
<td>0.30***</td>
</tr>
<tr>
<td>$\sigma_\beta$</td>
<td>(0.15)</td>
<td>(0.09)</td>
<td>(0.27)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4.27***</td>
<td>4.27***</td>
<td>4.27***</td>
<td>4.27***</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1.55***</td>
<td>1.55***</td>
<td>1.55***</td>
<td>1.55***</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>(0.15)</td>
<td>(0.15)</td>
<td>(0.15)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>N</td>
<td>150</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>-19471</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$. Coefficients are transformed back to the original scale. A treatment effect is the (partial) effect of moving from the baseline treatment to the corresponding treatment on the median parameter value. Standard errors are calculated using the sandwich estimator and treating all of each subject's choices as a single super observation. The entries for $\sigma$ are the standard deviations of the untransformed normal distributions of the random coefficients.

that we can reject the true parameters being the mean of the distribution represented by the recovered parameters, we can observe that $\text{MSE (bias}^2 + \text{variance})$ is quite small in these cases. For $\sigma_\beta, \sigma_\gamma$, the bias could be arising due to the unbounded nature of log-normal distribution.

7 Alternative Specifications of the Distributions of Social Preference Parameters and Identification

As an alternative to the log-normal specification of $\beta, \gamma$, in this appendix we explored a logistic-normal specification and its empirical identifiability. Both specifications are theoretically identified. Table 8 presents estimation results from logistic-normal specification using the data from strategy experiments. We use these estimates to do the empirical identification exercise as described in Appendix 6. The results from the empirical identification exercise are presented in Table 9. We conclude from the large bias in identified parameters in Table 9 that this specification has significant problems with empirical identification given our design and sample size.
Table 9: Empirical Identification of Logistic-Normal Specification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True</th>
<th>Recovered†</th>
<th>Bias</th>
<th>T-stat</th>
<th>Cohen’s $d$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_\beta$</td>
<td>-1.87 (13.34)</td>
<td>-1.23 (22.99)</td>
<td>0.63</td>
<td>13.35***</td>
<td>1.88</td>
<td>0.51</td>
</tr>
<tr>
<td>$\sigma_\beta$</td>
<td>5.29</td>
<td>6.84</td>
<td>1.56</td>
<td>8.032***</td>
<td>1.14</td>
<td>4.34</td>
</tr>
<tr>
<td>$\mu_\gamma$</td>
<td>-0.25 (43.83)</td>
<td>-0.039 (48.92)</td>
<td>0.21</td>
<td>3.36***</td>
<td>0.48</td>
<td>0.23</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>4.28</td>
<td>4.38</td>
<td>0.10</td>
<td>0.51</td>
<td>0.072</td>
<td>1.99</td>
</tr>
<tr>
<td>$\mu_\omega$</td>
<td>-0.83 (0.30)</td>
<td>-0.98 (0.27)</td>
<td>-0.15</td>
<td>-7.14***</td>
<td>-1.01</td>
<td>0.043</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>1.55</td>
<td>1.56</td>
<td>0.018</td>
<td>0.94</td>
<td>0.13</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Notes:
1. *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$
2. In brackets transformed median value

Table 10: Types in P1 and P2 Across Different Levels of MPCR

<table>
<thead>
<tr>
<th>Type</th>
<th>P1-Low</th>
<th>P1-High</th>
<th>P2-Low</th>
<th>P2-High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Riders</td>
<td>20 (27%)</td>
<td>16 (21%)</td>
<td>34 (45%)</td>
<td>20 (27%)</td>
</tr>
<tr>
<td>Conditional Cooperators</td>
<td>45 (60%)</td>
<td>44 (59%)</td>
<td>26 (35%)</td>
<td>43 (57%)</td>
</tr>
<tr>
<td>Full Contributors</td>
<td>7 (9%)</td>
<td>10 (13%)</td>
<td>11 (15%)</td>
<td>9 (12%)</td>
</tr>
<tr>
<td>Noisy Contributors</td>
<td>3 (4%)</td>
<td>5 (7%)</td>
<td>4 (5%)</td>
<td>3 (4%)</td>
</tr>
<tr>
<td>Total</td>
<td>75(100%)</td>
<td>75 (100%)</td>
<td>75(100%)</td>
<td>75(100%)</td>
</tr>
</tbody>
</table>

8 Distribution of Types Based on Classification using LCP

Table 10 presents the distribution of types in P1 and P2 experiments at different levels of MPCR.

9 Belief Formation in Repeated Games

We used the regression approach of Fischbacher and Gächter (2010) to study belief formation in repeated games. Regressions are conducted with the belief in a given round as the dependent variable and the belief from the previous round and the average contribution of other members of the group in the previous round as explanatory variables. Table 11 presents the regression estimates. Model (1) presents results for all sessions, whereas Models (2) and (3) report results for sessions with finitely repeated and indefinitely repeated games respectively. In all cases, the coefficient on the constant is insignificant and the sum of the coefficients on the belief and average contribution of others in the previous round is not different from one (all sessions: $F(1, 9) = 0.44, p = 0.522$; finitely repeated sessions: $F(1, 3) = 0.01, p = 0.944$; indefinitely repeated sessions: $F(1, 5) = 0.89, p = 0.388$). Beliefs in a given round are updated as a weighted average of the beliefs in the previous round and average of the others’ contributions in the previous round. The results are consistent with the findings in Fischbacher and Gächter (2010), and they use Strangers’ matching in the experiments. The regression coefficients are not different across the finitely and indefinitely repeated games (Chow-test, $F(3, 9) = 0.04, p = 0.990$). Thus, we
conclude that belief formation is identical across finitely and indefinitely repeated games.

10 Individual Level Imputations of Social Preference Parameters

In our estimations Table 3, we used only covariates for treatment effects. For a treatment, the density \( f(\eta_i|X_i, \hat{\delta}, \hat{\Sigma}, g_\eta) \) represents the population level distribution of \( \eta_i \) for treatments with characteristics given in \( X_i \). Using this distribution as a “prior” one can infer the individual level “posterior” distribution of \( \eta_i \) of an individual in a treatment with the characteristics \( X_i \) by conditioning on his/her observed choices. Say in treatment \( Z \) defined by covariates \( X_i \), observed individual level data is \( \{C_Z, O_Z\} \), where \( C_Z \) are the individual’s contributions in a sequence of decision situations faced in the treatment \( Z \) and \( O_Z \) are average contributions of other group members or beliefs about average contributions of other group members in those decision situations. \( Z \) can be one of the four experiments P1,P2,R1,R2. The posterior density \( h(\eta_i|X_i, C_Z, O_Z, \hat{\delta}, \hat{\Sigma}, g_\eta) \) that is obtained by updating the population level prior using the individual level data reflects all available information about structural parameters \( \eta_i \in \{\beta, \gamma, \omega\} \) of an individual. In the following, to keep notation less cluttered, we drop the term \( O_Z \) from posterior computations since it is not a non-degenerate random variable.

The computation of posterior density and its expectation is motivated using Baye’s rule (Revelt & Train, 2000). By using Baye’s rule one obtains:

\[
h(\eta_i|X_i, C_Z, \hat{\delta}, \hat{\Sigma}, g_\eta) = \frac{P(C_Z|\eta_i)f(\eta_i|\hat{\delta}, \hat{\Sigma}, g_\eta)}{P(C_Z|\hat{\delta}, \hat{\Sigma}, g_\eta)} \tag{7}
\]

We illustrate how the population level priors and then the posteriors conditioned on individual choices look like by using the choices of three participants in P1 experiment with low MPCR. The top left Panel in Figure 12 shows the contribution schedules of three participants S1, S2 and S3. participant S1 has a negatively sloped contribution schedule and thus his/her behavior cannot be explained as an equilibrium behavior using any

### Table 11: Beliefs in Repeated Games

<table>
<thead>
<tr>
<th>Belief, ( t )</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belief, ( t-1 )</td>
<td>0.38***</td>
<td>0.39***</td>
<td>0.32***</td>
</tr>
<tr>
<td>Average Others’ Contribution, ( t-1 )</td>
<td>0.62***</td>
<td>0.62***</td>
<td>0.62***</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.026</td>
<td>0.049</td>
<td>-0.056</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Obs</th>
<th>1920</th>
<th>720</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.87</td>
<td>0.85</td>
<td>0.88</td>
</tr>
</tbody>
</table>

*p < 0.1, ** p < 0.05, *** p < 0.01

Robust standard errors clustered at session level in parentheses
values of social preference parameters. Participant S2 looks like a conditional cooperator. Finally, participant S3’s behavior is close to that of a full cooperator.

Since we are considering choices in P1 experiment with low MPCR, we consider the population level distribution of structural parameters in P1 with low MPCR as the prior distribution. Marginal prior distributions are plotted using black in Panels that correspond to the structural parameters \(\beta, \gamma, \omega\) in Figure 12. The 10% quantile and 90% quantile of prior \(\beta\) marginal are 1.25, 1216.92 respectively. The 10% quantile and 90% quantile of prior \(\gamma\) marginal are 2.01, 129.54 respectively. Both marginal priors \(\beta, \gamma\) are very dispersed with medians at 39.10 and 16.09. Similarly, for the prior \(\omega\) marginal the 10% quantile and 90% quantile are 0.09% and 76%, and its median is 5.1%.

The marginal posterior distributions of social preference parameters of participant S1 are almost indistinguishable from that of the prior marginals. The 10% quantile and 90% quantile of posterior \(\beta\) marginal are 0.78, 840.66 respectively. The 10% quantile and 90% quantile of posterior \(\gamma\) marginal are 1.90, 127.39 respectively. The marginals of posterior social preference parameters are nearly as dispersed as that of the priors and are only slightly tighter. This is because the responses of participant S1, as shown in top-left Panel, cannot be explained as being consistent with the equilibrium contributions for any values of social preference parameters. In other words, the responses of S1 carry a very little information about social preference parameters. The marginal posterior distribution of the random choice probability of S1 indicates that the participant’s choices can be classified as almost random. S1’s random choice propensity parameters is greater than 0.8 with 90% probability. Since S1’s responses cannot explained using social preferences model, almost all the likelihood of the data comes from the random choice model and thus the posterior parameter \(\omega\) is very large for S1. S1’s behavior is probably driven by something different than that of social preference model.

The participant S2 made choices in P1 that are more in line with that of a conditional cooperator. Not surprisingly, her marginal posteriors of social preference parameters reflect it very clearly, their median values show that she is a conditional cooperator. Both marginals of social preference parameters (in green) are quite tight compared to that of marginal prior distributions (in black). The 10% quantile and 90% quantile of posterior \(\beta\) marginal are 15.58, 21.61 respectively. The 10% quantile and 90% quantile of posterior \(\gamma\) marginal are 2.12, 25.18 respectively. Indeed, for any values of \(\beta, \gamma\) posteriors in between 10% and 90% quantiles, S2’s best response is to match the average contribution of the group, i.e. she will behave as a conditional cooperator in the equilibrium. The random choice propensity of S2 is quite small and below 9.7% with probability of 90% probability.

S3’s choices in P1 are close to that of a full cooperator. His median posterior social preference parameters indicate that he will contribute the full endowment in equilibrium. Like that of S2, S3’s posteriors of the social preference parameters (in maroon) are very tight compared to the corresponding priors. For S3, the 10% quantile and 90% quantile of the posterior \(\beta\) marginal are 16.44, 31.96 respectively. The 10% quantile and 90% quantile of the posterior \(\gamma\) marginal are 0.18, 1.53 respectively. His altruistic parameter \(\beta\) is large and the envy parameter \(\gamma\) is quite small indicating his inclination towards full cooperation. Given his posterior \(\beta\), the probability that S3 can act as a free rider is less
than 0.01% which is extremely small (S3 will behave as if a free rider in P1 with low MPCR if 0.07β − 0.6 < 0). For the largest chunk of joint distribution of posterior β, γ, S3 will behave like a full cooperator. The random choice propensity of S3 is very small and below 6.7% with probability of 90% probability.

Thus for all the three structural parameters, conditioning on the individual level choices makes the implied parameter ranges much tighter. We find that the individual level responses are generally very informative about individual preference parameters, except, as expected, in the cases where choices suggest that the participants choices are not consistent with the theory of social preferences or probably completely random. Hence, by conditioning on individual choices, we can arrive at social preference parameter and noise parameter values that can accurately describes the behavior of a participant.

Expectations of the posterior distributions of structural parameters can be used to characterize how a given individual would on average behave in a given treatment. This approach is powerful in that expected values of structural parameters computed by conditioning on say choices in a strategy game can be used to “predict” the likelihood of choices in a repeated game. Thus one can evaluate the efficacy of information obtained by observing the behavior of an individual in one of the treatments to predict the behavior in a new or different treatment.

It is straightforward to compute the expectation of posterior of a parameter ηᵢ starting from Equation 7 using:
\[ E(\eta|X_i, C_Z, \hat{\delta}, \hat{\Sigma}, g_\eta) = \int \eta h(\eta|X_i, C_Z, \hat{\delta}, \hat{\Sigma}, g_\eta) \]
\[ = \int \eta P(C_Z|\eta) f(\eta|\hat{\delta}, \hat{\Sigma}, g_\eta) \]
\[ = \frac{\int \eta P(C_Z|\eta) f(\eta|\hat{\delta}, \hat{\Sigma}, g_\eta)}{P(C_Z|\hat{\delta}, \hat{\Sigma}, g_\eta)} \]
\[ \tag{8} \]

Equation 8 is approximated using simulation:
\[ \tilde{E}(\eta|X_i, C_Z, \hat{\delta}, \hat{\Sigma}, g_\eta) = \sum_r \eta^r P(C_Z|\eta^r) \]
\[ \frac{P(C_Z|\eta^r)}{P(C_Z|\eta^r)} \]
\[ \tag{9} \]

Where \( \eta^r \) is the \( r \)th draw from the population prior density \( f(\eta|X_i, \hat{\delta}, \hat{\Sigma}, g_\eta) \). In computing expectations of posterior distributions, we used 10,000 Halton draws from the prior distribution for each individual to obtain accurate posterior estimates.